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RESEARCH MEMORANDUM

ANALYSIS OF LIMITATIONS IMPOSED ON ONE-SPOOL TURBOJET-ENGINE DESIGNS BY COMPRESSORS AND TURBINES AT FLIGHT MACH NUMBERS OF 0, 2.0, AND 2.8

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NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

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SUMMARY

A design-point analysis of one-spool turbojet engines was made to determine the relations among engine, compressor, and turbine design parameters in order to ascertain the primary limitations on turbojet-engine design. In particular, the analysis is used to reveal the manner in which the design problem is affected by variations in flight Mach number. Sea-level operation and flight at Mach numbers of 2.0 and 2.8 in the stratosphere are considered.

The results of this analysis show that compressor aerodynamics is a primary constraint only for design-point operation under static sea-level conditions. For all three flight conditions considered, turbine aerodynamics is critical for one-stage turbines; whereas, it is critical for two-stage turbines only at the 2.8 flight Mach number in the stratosphere. For flight at 2.0 Mach number in the stratosphere, turbine blade centrifugal stress for both one- and two-stage turbines becomes a limiting factor, and it is a primary constraint for Mach 2.8 engine designs. At the 2.8 flight Mach number, the compressor that can be driven by a two-stage turbine can be so conservative as to possess high potential for equivalent overspeed capacity. Such a characteristic should prove useful in providing good low flight Mach number operation of constant-geometry engines. Raising turbine rotor-inlet relative Mach number from 0.8 to 1.0 in one-stage turbines designed for 2.8 flight Mach number yields increases up to 8 percent in weight-flow capacity, provided the turbine efficiency does not deteriorate.

INTRODUCTION

In the process of turbojet-engine design, the interdependence of the engine components makes difficult the problem of selecting, for a set of design factors, values that effectively utilize all the components. If, for example, selection of rotational speed to yield a good compressor design is not tempered by a simultaneous consideration of the effect of rotational speed on turbine design, a large or multistage turbine critical in aerodynamic design may be required. Such a possibility can be avoided by selecting a series of values for design factors and, for each set of values, investigating the design of each component. From a compilation of such design information, a satisfactory design condition can then be selected.

An analysis relating these factors has been developed at the NACA Lewis laboratory. This design-point analysis of one-spool turbojet engines relates the following factors: 3275

(1) Engine design parameters:

Flight Mach number and altitude
Engine temperature ratio
Compressor pressure ratio

(2) Compressor design parameters:

Compressor-inlet relative Mach number (assuming no inlet guide vanes)
Compressor equivalent blade tip speed
Centrifugal stress in first row of compressor rotor blades

(3) Turbine design parameters:

Aerodynamic design limits
Centrifugal stress in turbine rotor blades
Turbine hub-tip radius ratio
Equivalent weight flow per unit turbine frontal area

This report has as its goals (1) to present the method of analysis, (2) to present design charts for several applications, and (3) to investigate the primary limitations on turbojet-engine design and, in particular, to show the manner in which the design problem is affected by the increase in compressor-inlet temperature resulting from an increase in flight Mach number.

The analysis developed herein is presented in the form of design-point charts, two sets for compressors and one set for turbines. Because full-scale compressors have been successfully operated with rotor-inlet relative Mach numbers as high as 1.2, this value is accepted herein as a limit. Compressor-inlet relative Mach numbers up to 1.4 are considered, however, because reference 1 reports that high efficiency has been obtained experimentally at this Mach number in a one-stage compressor without inlet guide vanes. Engine temperature ratios from 2.0 to 4.0 and flight conditions ranging from a flight Mach number of 0 at sea level to 2.8 in the stratosphere are considered. Both one-stage and two-stage turbines are investigated. The term "conservative" is applied to turbines limited by 0.6 relative Mach number at the rotor inlet; the term "high-output" is applied to turbines limited by 0.8 and by 1.0 relative Mach numbers at the rotor inlet.

Because preselected aerodynamic limits were used in preparation of the turbine charts, the results obtained herein represent extremes, or outer bounds, for this range of turbojet-engine designs. The complication introduced by the requirements of off-design operation is not considered.

ANALYSIS AND CONSTRUCTION OF CHARTS

The present analysis is formulated around a certain parameter $wU_t^2/A_t\delta_1'\sqrt{\theta_1'}$, herein dubbed parameter e, which is of great utility in relating compressors and turbines. The symbols used are listed in appendix A. Parameters e for compressors and turbines are as follows:

$$\left.\begin{aligned} &\text{Compressor:} \quad \frac{w_C U_{t,C}^2}{A_C\delta_1'\sqrt{\theta_1'}} = \frac{w_C\omega^2 r_{t,C}^2}{\pi r_{t,C}^2\delta_1'\sqrt{\theta_1'}} = \frac{w_C\omega^2}{\pi\delta_1'\sqrt{\theta_1'}} \\ &\text{Turbine:} \quad \frac{w_T U_{t,T}^2}{A_T\delta_1'\sqrt{\theta_1'}} = \frac{w_T\omega^2 r_{t,T}^2}{\pi r_{t,T}^2\delta_1'\sqrt{\theta_1'}} = \frac{w_T\omega^2}{\pi\delta_1'\sqrt{\theta_1'}} \end{aligned}\right\} \qquad (1)$$

Figure 1 shows the location of the numerical stations in the engine.

Because compressor and turbine weight flows bear the relation

$$w_T = (1+f_3)(1-b)w_C \qquad (2)$$

parameters e for compressors and turbines are related by

$$\frac{w_T U_{t,T}^2}{A_T\delta_1'\sqrt{\theta_1'}} = (1+f_3)(1-b)\,\frac{w_C U_{t,C}^2}{A_C\delta_1'\sqrt{\theta_1'}} \qquad (3)$$

For a specified fuel-air ratio f_3 and compressor bleed b, parameters e for compressors and turbines are related by a constant factor. If the fuel-air ratio and bleed are small, little error is made in assuming that parameters e for compressors and turbines are equal.

Inasmuch as the results of references 2 and 3 are extended and supplemented to form the present analysis, the same assumptions used in those references are again applied. These assumptions are reviewed in appendix B along with a list of constants.

Compressor Charts

Parameter e, from the standpoint of the compressor, is simply the product of compressor equivalent weight flow per unit compressor frontal area, hereafter called equivalent specific air flow and denoted by $w_C\sqrt{\theta_1'}/A_C\delta_1'$, and the square of compressor equivalent blade tip speed $(U_{t,C}/\sqrt{\theta_1'})^2$. In the entire compressor analysis presented herein, it is assumed that the compressors have no inlet guide vanes. With this assumption, the equation developed for parameter e in appendix C is 3275

$$e = \frac{w_C U_{t,C}^2}{A_C\delta_1'\sqrt{\theta_1'}} = 2116\sqrt{518.7}\sqrt{R(\gamma g)^3}\,\frac{\left(\frac{V}{a}\right)_1\left[M_1^2 - \left(\frac{V}{a}\right)_1^2\right]}{\left[1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2\right]^{\frac{3\gamma-1}{2(\gamma-1)}}}\left[1 - \left(\frac{r_h}{r_t}\right)_C^2\right] \tag{C11}$$

and is plotted in figure 2 by assuming a range of values of $(V/a)_1$. In this chart, parameter e $(w_C U_{t,C}^2/A_C\delta_1'\sqrt{\theta_1'})$ is plotted against compressor equivalent blade tip speed $U_{t,C}/\sqrt{\theta_1'}$ for three values of compressor rotor-inlet relative Mach number M_1 and two values of compressor hub-tip radius ratio $(r_h/r_t)_C$.

The dotted line in figure 2 joins the maximum points of the curves of constant compressor rotor-inlet relative Mach number. Maximum parameter e for a given Mach number corresponds to minimum Mach number for a given parameter e. The dotted line, therefore, is the locus of minimum values of compressor rotor-inlet relative Mach number for given values of parameter e. For various limits on compressor rotor-inlet relative Mach number M_1, along this dotted line compressors may be designed with maximum values of e. Compressor rotor-inlet relative Mach number could be reduced by turning the entering flow in the direction of rotor rotation by means of inlet guide vanes and thereby producing smaller values of inlet relative tangential velocity.

An additional abscissa scale is presented in figure 2 showing the level of compressor centrifugal blade stress. The parameter σ_C/θ_1' can be thought of as an equivalent blade centrifugal stress. This parameter was calculated from the relation

$$\frac{\sigma_C}{\theta_1'} = \frac{\Gamma_C \psi_C}{2g(144)} \left(\frac{U_{t,C}}{\sqrt{\theta_1'}}\right)^2 \left[1 - \left(\frac{r_h}{r_t}\right)_C^2\right] \tag{4}$$

3275 which is derived from equation (8) of reference 4.

In order to obtain accurate values of the various compressor parameters at the peaks of the Mach number curves in figure 2, the expression for parameter e has been differentiated with respect to compressor-inlet Mach number $(V/a)_1$ and set equal to zero in appendix D. The term "maximized parameter e" refers to values of parameter e read from the dotted line of figure 2. The following equation was derived for compressor-inlet Mach number for maximized parameter e:

$$\left(\frac{V}{a}\right)_1 = \sqrt{\frac{1}{2}\left[\gamma M_1^2 + 3 - \sqrt{(\gamma M_1^2 + 3)^2 - 4M_1^2}\right]} \tag{D5}$$

Maximized parameter e can be calculated by substitution of equation (D5) into equation (C11).

The corresponding equivalent specific air flow for maximized parameter e is

$$\frac{w_C\sqrt{\theta_1'}}{A_C\delta_1'} = \frac{2116}{\sqrt{518.7}}\sqrt{\frac{\gamma g}{R}} \frac{\left(\frac{V}{a}\right)_1}{\left[1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}} \left[1 - \left(\frac{r_h}{r_t}\right)_C^2\right] \tag{C10}$$

where $(V/a)_1$ is specified by equation (D5). Likewise, the compressor equivalent blade tip speed for maximized parameter e is

$$\frac{U_{t,C}}{\sqrt{\theta_1'}} = \sqrt{518.7}\,\sqrt{\gamma g R}\,\sqrt{\frac{M_1^2 - \left(\frac{V}{a}\right)_1^2}{1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2}} \tag{C6}$$

$(V/a)_1$ again being determined by equation (D5).

Figure 3 consists entirely of plots of compressor parameters for maximized parameter e. In figures 3(a) and (b), maximized parameter e $(w_C U_{t,C}^2/A_C \delta_1' \sqrt{\theta_1'})_{max}$ and equivalent specific air flow $w_C \sqrt{\theta_1'}/A_C \delta_1'$ are plotted against compressor rotor-inlet relative Mach number M_1 for two values of compressor hub-tip radius ratio $(r_h/r_t)_C$. In figure 3(c), compressor equivalent blade tip speed $U_{t,C}/\sqrt{\theta_1'}$ and compressor-inlet Mach number $(V/a)_1$ are plotted against M_1. As in figure 2, the compressor-inlet velocity was assumed axial; that is, the compressors were assumed to have no inlet guide vanes. 3275

Turbine Charts

By continuity and stress considerations in the turbine, the following equation for parameter e is derived in appendix E:

$$\frac{w_T U_{t,T}^2}{A_T \delta_1' \sqrt{\theta_1'}} = \frac{2116}{\sqrt{518.7}}\, 288 \sqrt{\frac{2kg^3}{(k+1)R}} \left(\frac{\rho V_x}{\rho' a_{cr}'}\right)_{7,m} \frac{\sigma_T}{\theta_1' \Gamma_T \psi_T} \times \frac{\left[1 - \frac{\gamma}{\gamma-1}\,\frac{k-1}{k}\,\frac{\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}}-1}{\eta_T \eta_C \frac{T_3'}{T_1'}}\right]^{\frac{k}{k-1}} \frac{p_2'}{p_1'}\,\frac{p_3'}{p_2'}}{\left[1 - \frac{\gamma}{\gamma-1}\,\frac{k-1}{k}\,\frac{\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}}-1}{\eta_C \frac{T_3'}{T_1'}}\right]^{\frac{1}{2}} \sqrt{\frac{T_3'}{T_1'}}} \qquad \text{(E9)}$$

Equation (E9) shows that, for a given turbine blade centrifugal stress σ_T, parameter e is a function of θ_1', compressor pressure ratio p_2'/p_1', engine temperature ratio T_3'/T_1', and turbine-exit specific-weight-flow parameter $(\rho V_x/\rho' a_{cr}')_{7,m}$, if compressor and turbine adiabatic efficiencies are assumed constant. But for the variable θ_1', which is a function of flight Mach number and altitude, the entire analysis presented herein could be presented independent of flight conditions. The analysis, however, can be generalized to apply to all flight conditions if equivalent turbine blade centrifugal stress σ_T/θ_1' be considered as an independent variable in equation (E9). The relation between parameter e and turbine blade centrifugal stress is thus independent of the turbine design (e.g., number of stages or aerodynamic limits), except for the turbine-exit specific-weight-flow parameter $(\rho V_x/\rho' a_{cr}')_{7,m}$. A value of 0.448 was assumed for this parameter for all the conservative turbines considered, that is, for turbines designed with an exit axial-velocity ratio $(V_x/a_{cr}')_7$ of 0.5. This value of 0.448 was determined from the relation

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$$\left(\frac{\rho V_x}{\rho' a_{cr}'}\right)_{7,m} = \left[1 - \frac{k-1}{k+1}\left(\frac{V}{a_{cr}'}\right)_{7,m}^2\right]^{\frac{1}{k-1}} \left(\frac{V_x}{a_{cr}'}\right)_7 \qquad (5)$$

neglecting exit tangential velocity ratio $(V_u/a_{cr}')_{7,m}$. All such conservative turbines considered in the present analysis are two-stage designs. For all the one-stage turbines and the high-output two-stage turbines considered herein, a value of 0.562 was assumed for the parameter $(\rho V_x/\rho' a_{cr}')_{7,m}$. This value was obtained from equation (5) by again neglecting $(V_u/a_{cr}')_{7,m}$ and using a value of 0.7 for $(V_x/a_{cr}')_7$.

The turbine charts (fig. 4) consist of design performance maps of one- and two-stage turbines. These charts are constructed for three selected flight conditions: sea-level static, and 2.0 and 2.8 flight Mach numbers in the stratosphere. In all the turbine charts presented, equation (E9) was used to plot parameter e, $w_T U_{t,T}^2/A_T \delta_1' \sqrt{\theta_1'}$, against compressor pressure ratio p_2'/p_1' with lines of constant turbine blade centrifugal stress σ_T for selected values of flight Mach number M_0 and engine temperature ratio T_3'/T_1'. The discontinuities in the turbine blade centrifugal-stress curves in figures 4(a), (d), and (e) occur because the one-stage turbines are high-output designs, whereas the two-stage turbines are conservative designs. The blade centrifugal-stress curves of the conservative two-stage turbines are lowered relative to those of the one-stage turbines by the ratio of 0.448/0.562 (eq. (E9)). The charts of figure 4 can be made to apply to any particular flight condition if the values of stress as shown on the chart are multiplied by the ratio of θ_1' for the chart flight condition (0, 2.0, or 2.8 flight Mach number) to θ_1' for the particular flight condition.

The turbine charts were completed by the addition of lines of constant turbine-limited specific weight flow, which is the terminology to be used herein for $w_T \sqrt{\theta_1'}/A_T \delta_1'$, and turbine hub-tip radius ratio

$(r_h/r_t)_T$. These lines were obtained from the data presented in references 2 and 3. In these charts, regions to the left apply to one-stage turbines, and those to the right, to two-stage turbines.

A straight annulus was used in the calculations for all one-stage turbine designs presented herein. For the two-stage turbines, an isentropic annular area ratio of 1.0 was assumed for each rotor blade row in order to simplify the calculations. As a consequence of this arbitrary assumption for isentropic annular area ratio, divergences in actual annular area occur across each rotor blade row. These divergences are as follows: 3275

$$\frac{A_5}{A_4} = \exp\left[\frac{J(s_5 - s_4)}{R}\right]$$

and

$$\frac{A_7}{A_6} = \exp\left[\frac{J(s_7 - s_6)}{R}\right]$$

Table I summarizes the engine parameters and turbine aerodynamic parameters selected for presentation in the turbine charts. Except for the turbine blade centrifugal-stress lines, charts having the same engine temperature ratio and turbine aerodynamic parameters are identical. The values of engine temperature ratio were selected to give turbine-inlet temperatures, at the flight Mach numbers considered, of approximately 2000°, 2500°, and 3000° R. The turbine charts can be applied to any Mach number desired with appropriate revision of the turbine blade centrifugal-stress curves (see eq. (E9)).

GENERAL CONSIDERATIONS IN ENGINE DESIGN

Before making a detailed examination of particular points on the turbine charts, it is desirable to consider some general characteristics of the curves presented. A consideration of desirable engine features will eliminate certain regions of the turbine charts.

Turbine-Stress Curves

The peaks of the stress curves on the turbine charts represent (1) maximum equivalent weight flow per unit turbine annular area, (2) maximum equivalent weight flow per unit afterburner frontal area for a given afterburner-inlet Mach number, and (3) minimum turbine blade centrifugal stress for a given compressor-inlet relative Mach number and compressor equivalent blade tip speed. These three items, which hold true for any given flight Mach number, altitude, and engine temperature ratio, are explained as follows.

(1) Maximum equivalent weight flow per unit turbine annular area. - Equation (E3) shows that, for any given turbine blade centrifugal stress, under the conditions enumerated previously a maximum parameter e corresponds to a maximum value of $p_7'/\sqrt{T_7'}$. Division of equation (E2) by
3275 the factor $\left[1 - (r_h/r_t)_T^2\right]$ yields equivalent weight flow per unit turbine annular area $w_T\sqrt{\theta_1'}/A_7\delta_1'$ and shows that this parameter is directly proportional to $p_7'/\sqrt{T_7'}$.

(2) Maximum equivalent weight flow per unit afterburner frontal area. - That peaks on the turbine-stress curves correspond to this item, for the engine conditions that are assumed constant, is revealed by con-
CD-2 sidering the continuity equation at the afterburner inlet. The weight flow at the afterburner inlet is

$$w_8 = \rho_8 V_8 A_A$$

which, when expanded, becomes

$$\frac{w_8\sqrt{\theta_1'}}{A_A\delta_1'} = \frac{2116}{\sqrt{518.7}}\sqrt{\frac{kg}{R}}\left(\frac{V}{a}\right)_8\left[1 + \frac{k-1}{2}\left(\frac{V}{a}\right)_8^2\right]^{-\frac{k+1}{2(k-1)}}\frac{p_8'}{p_7'}\,\frac{p_7'}{\sqrt{T_7'}}\sqrt{\frac{T_7'}{T_8'}}\,\frac{\sqrt{T_1'}}{p_1'} \tag{6}$$

In equation (6), T_8' equals T_7', and the ratio p_8'/p_7' can be assumed constant. For a given afterburner-inlet Mach number $(V/a)_8$, therefore, equivalent weight flow per unit afterburner frontal area is proportional to $p_7'/\sqrt{T_7'}$, which, in turn, is proportional to parameter e in accordance with equation (E3).

(3) Minimum turbine stress for given compressor-inlet relative Mach number and equivalent blade tip speed. - A given compressor-inlet relative Mach number and equivalent blade tip speed specify a value of parameter e (fig. 2). Item (3) then becomes apparent by observing in figure 4 that the minimum turbine blade centrifugal stress for a particular value of parameter e must be that stress line which peaks at the particular parameter e.

Compressor Pressure Ratio for Minimum Specific Fuel Consumption for Afterburning Engines

In all the turbine charts, each of which is constructed for a constant value of turbine-inlet temperature, the point of minimum specific fuel consumption for afterburning engines with a given afterburner-exit temperature lies slightly to the left of the stress-curve peaks. Specific fuel consumption, assuming complete expansion in the exhaust nozzle, is 3275

$$sfc = \frac{3600g(f_3+f_9)}{(1+f_3+f_9)V_{10}-V_0} \tag{7}$$

where

$$V_{10} = C_n \sqrt{2gR \frac{k}{k-1} T_9' \left[1 - \left(\frac{p_0}{p_9'}\right)^{\frac{k-1}{k}}\right]} \tag{8}$$

and

$$\frac{p_0}{p_9'} = \frac{p_0}{p_0'} \frac{p_0'}{p_1'} \frac{p_1'}{p_7'} \frac{p_7'}{p_9'} \tag{9}$$

and T_9' is the afterburner-exit temperature. The first two pressure ratios on the right side of equation (9) are functions of the flight Mach number and inlet performance. The ratio p_7'/p_9' is assumed constant as compressor pressure ratio is varied. For a given afterburner-exit temperature, a maximum value for the ratio p_7'/p_1' corresponds to a maximum value of V_{10} by equation (8), and in turn to a minimum value of sfc by equation (7). The value of compressor pressure ratio p_2'/p_1' corresponding to a maximum value of p_7'/p_1' was determined by plotting p_7'/p_1' against p_2'/p_1' and noting the value of p_2'/p_1' at which p_7'/p_1' peaks. The ratio p_7'/p_1' can be expressed

$$\frac{p_7'}{p_1'} = \frac{p_7'}{p_3'} \frac{p_3'}{p_2'} \frac{p_2'}{p_1'} \tag{E5}$$

in which p_7'/p_3' is given by equation (E8) and p_3'/p_2' is assumed constant. Table II lists for a given afterburner-exit temperature the compressor pressure ratios corresponding to minimum specific fuel consumption for each engine temperature ratio presented in the turbine charts (fig. 4). These compressor pressure ratios are designated by arrows on the turbine charts.

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Considerations of Engine Component Sizes

The component of an engine that limits engine frontal area should receive special attention towards reducing its frontal area, especially for engines mounted in a nacelle. In the design of those components not limiting engine frontal area, the frontal areas are less important; this fact permits emphasis of other desirable characteristics in these particular components. For example, in the event that compressor size is not limiting, it might be desirable to increase compressor frontal area in the interest of enhancing the off-design operation. The special attention devoted to the frontal area of the limiting component might affect the choice of values for other engine design variables.

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Reductions of component frontal areas are brought about by increases in component specific weight flows. The engine component equivalent weight flows per unit component frontal area have the following trends with increasing compressor pressure ratio: (1) that of the primary combustor increases (for a given equivalent specific air flow and combustor-inlet Mach number); (2) that of the turbine decreases (as shown in fig. 4); and (3) that of the afterburner increases to a maximum at the peak of a stress curve in figure 4 and then decreases as compressor pressure ratio increases further. Figure 4, therefore, since it also presents compressor pressure ratio as one of its parameters, indicates the general direction that a change in engine design variables should take to effect a reduction in the frontal area of a component limiting nacelle frontal area.

Furthermore, on the turbine charts a line of constant hub-tip radius ratio corresponds to a constant ratio of afterburner-to-turbine frontal area for a given afterburner-inlet Mach number. This fact can be shown as follows:

$$(\rho V_x)_{7,m} A_7 = (\rho V_x)_{8,m} A_A$$

which can be written

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{7,m}\left[1-\left(\frac{r_h}{r_t}\right)_T^2\right]=\frac{p_8'}{p_7'}\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{8,m}\frac{A_A}{A_T} \tag{10}$$

On a given turbine chart, $(\rho V_x/\rho' a'_{cr})_{7,m}$ has a value of either 0.448 or 0.562, depending upon whether the turbine designs are conservative or high-output. The parameter $(\rho V_x/\rho' a'_{cr})_{8,m}$ is a function of afterburner-inlet Mach number by 3275

$$\left(\frac{\rho V_x}{\rho' a'_{cr}}\right)_{8,m}=\sqrt{\frac{k+1}{2}}\left(\frac{V_x}{a}\right)_{8,m}\left[1+\frac{k-1}{2}\left(\frac{V}{a}\right)_{8,m}^2\right]^{-\frac{k+1}{2(k-1)}} \tag{11}$$

For an afterburner-inlet Mach number of 0.25 (500 ft/sec afterburner-inlet velocity at 1766° R stagnation temp.), a value of 0.98 for p_8'/p_7', and wherever on the turbine charts $(\rho V_x/\rho' a'_{cr})_{7,m}$ has a value of 0.562, a turbine hub-tip radius ratio value of 0.74 corresponds to equal turbine and afterburner frontal areas. On the turbine charts to the right of such a hub-tip radius-ratio line (values exceeding 0.74), turbine frontal area is greater than afterburner frontal area.

The turbine charts for flight Mach numbers of 2.0 and 2.8 in the stratosphere (figs. 4(b) to (i)) show that one-stage turbines have low weight-flow capacity at a 30,000-pound-per-square-inch turbine stress. Whereas these turbine-limited specific weight flows are of the order of 20 lb/(sec)(sq ft), the equivalent specific air flows of the compressor, at the same parameter e, can be considerably higher. Such a difference in relative sizes of compressor and turbine in itself is of no primary concern. If the turbine size limits nacelle frontal area, it would probably be advisable to select a design point on the turbine chart at reduced compressor pressure ratio and possibly increased turbine stress to exploit more fully the potentialities of the compressor. The decrease in nacelle frontal area will result in lower nacelle drag, which might well offset the increase in specific fuel consumption occasioned by further deviating from the compressor pressure ratio for minimum specific fuel consumption.

Critical Comment

It can be concluded from the preceding discussion that the region of primary interest on the turbine charts is generally confined to that to the left of the peaks of the turbine blade centrifugal-stress curves.

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RESULTS AND DISCUSSION

In the following discussion, the primary parameters considered in ascertaining engine design limitations are compressor-inlet relative Mach number, turbine Mach numbers, and turbine blade centrifugal stress. Parameter e, it will be seen, is of great utility in determining these limitations. Parameter e has the following significance: Use of a high value of parameter e for a specified compressor pressure ratio and engine temperature ratio results in a high value of turbine-limited specific weight flow for turbines of given aerodynamic limits; this high turbine weight-flow capacity is obtained, however, at the expense of increased compressor-inlet relative Mach number and increased turbine blade centrifugal stress. Or, alternatively, for a given turbine-limited specific weight flow, use of a high value of parameter e permits relaxing the turbine aerodynamic limits, a procedure that may be accompanied by improvement in turbine efficiency. Throughout this discussion, it will be assumed for simplicity that parameter e is the same for compressor and turbine.

For a compressor of the class used in the J35 and J47 turbojet engines, the value of parameter e is about 23 million lb/sec^3. Figure 3(a) shows that this value of parameter e is obtainable with a compressor-inlet relative Mach number as low as 0.93. Current laboratory compressor design techniques permit designing for compressor-inlet relative Mach numbers of 1.2 with confidence. Figure 2(a) shows that the maximum value of parameter e at this 1.2 compressor-inlet relative Mach number is 44 million lb/sec^3. It is revealed by any of the turbine charts that such an increase in parameter e (from 23 to 44 million lb/sec^3) is accompanied by an increase in turbine-limited specific weight flow for a given compressor pressure ratio.

Sea-Level Static Designs

Turbine-inlet temperature of 2075° R. - In the region to the left in figure 4(a) are considered high-output one-stage turbines designed for static sea-level operation, or for flight at 1.28 Mach number in the stratosphere, at an engine temperature ratio of 4.0. Characteristics of conservative two-stage turbines designed for the same flight conditions are presented in the area to the right on this chart. Point 1 represents the J35-J47 class of turbine.

If the stress level of a noncooled one-stage turbine can be raised to the current limit of 30,000 pounds per square inch and parameter e to a value of 44 million lb/sec^3 (obtainable with a compressor-inlet relative Mach number of 1.2, compressor equivalent blade tip speed of 1153 ft/sec and compressor-inlet hub-tip radius ratio of 0.4), a turbine-limited specific weight flow of about 25 lb/(sec)(sq ft) can be achieved (point 2). Figure 2(a) shows that the compressor blade centrifugal stress is 42,000 pounds per square inch at the equivalent blade tip speed of 1153 feet per second. For a given value of parameter e, it is revealed on any of the turbine charts that higher values of compressor pressure ratio and lower values of turbine blade centrifugal stress (for designs to the left of the stress-curve peaks) are obtainable if the turbine-limited specific weight flow is decreased and the turbine hub-tip radius ratio is increased. If, by turbine-cooling, the limit on turbine blade centrifugal stress could be raised to 50,000 pounds per square inch, turbine aerodynamics would permit designing for a compressor pressure ratio of 7.25 and a turbine-limited specific weight flow of 32 lb/(sec)(sq ft) (point 3, fig. 4(a)). At this point on the turbine chart, however, parameter e reaches 77 million lb/sec^3, a value which figure 2 indicates is currently beyond the range of efficient compressor design. Therefore, for one-stage turbines designed for the static sea-level condition, the principal constraints are compressor and turbine aerodynamics. This conclusion stems from the observation that compressor-inlet relative Mach number becomes limiting at turbine blade centrifugal-stress levels of 30,000 pounds per square inch at a compressor pressure ratio of 6.25 in figure 4(a) (point 2).

The area to the right in figure 4(a) discloses that, without exceeding either the 44 million lb/sec^3 limit on parameter e or the 30,000-psi limit on turbine blade centrifugal stress, a conservative two-stage turbine can produce a turbine-limited specific weight flow of 33 lb/(sec)(sq ft) at a compressor pressure ratio of 7.25 (point 4). At a compressor pressure ratio of 12.0, a conservative two-stage turbine has a turbine-limited specific-weight-flow capacity of about 32 lb/(sec)(sq ft) at the 44 million lb/sec^3 limit on parameter e (point 5), if the turbine blade centrifugal stress is raised to 33,000 pounds per square inch. Since the two-stage turbines considered in figure 4(a) are conservative with respect to turbine aerodynamics, this particular chart discloses that, for sea-level static designs employing two-stage turbines, compressor aerodynamics is the most stringent constraint. For the sea-level static design condition, aerodynamically conservative two-stage turbines with moderate turbine blade centrifugal stress possess the high weight-flow and work capacities required.

The values of compressor pressure ratio listed in table II are for minimum specific fuel consumption of afterburning engines and are probably not suitable for use with low flight Mach number engine designs such as are considered in figure 4(a). The compressor pressure ratios for minimum specific fuel consumption of nonafterburning engines are considerably greater than those for afterburning engines; that is, greater than the value of 11.6 given in table II for an engine temperature ratio of 4.0. Use of compressor pressure ratios of 12 or greater in one-spool engines is probably limited primarily by compressor surging during engine off-design operation.

Critical comment. - Compressor aerodynamics is the most serious problem for engines designed for the sea-level static condition. In order to achieve acceptable weight-flow capacity without exceeding current turbine stress limits, a one-stage turbine must be critical in design. Conservative two-stage turbines have adequate weight-flow and work capacities. Turbine blade centrifugal stress is not a limiting factor in these engine designs. The compressor pressure ratio obtainable from a high-output one-stage turbine is considerably lower than that for minimum specific fuel consumption of afterburning engines.

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Mach 2.0 Designs in Stratosphere

Turbine-inlet temperature of 2106° R. - In the section GENERAL CONSIDERATIONS IN ENGINE DESIGN, it was observed that afterburning engines will generally be designed for compressor pressure ratios no greater than and frequently less than that which provides minimum specific fuel consumption. For an engine temperature ratio of 3.0 (2106° R turbine-inlet temp.), for which the chart in figure 4(b) is constructed, a compressor pressure ratio of 5.0 is therefore in a reasonable range. Point 6 is located at this compressor pressure ratio and at the 44 million lb/sec^3 limit on parameter e. Point 6 lies in the range of one-stage turbines, and the turbine blade centrifugal stress at this point is 50,000 pounds per square inch. The turbine-limited specific weight flow for the design of point 6 is 24.3 lb/(sec)(sq ft).

Point 7 simultaneously locates feasible one- and two-stage turbine designs at the same compressor pressure ratio as point 6, but at a turbine blade centrifugal stress reduced to 30,000 pounds per square inch. The one-stage turbine has a weight-flow capacity of 18.5 lb/(sec)(sq ft), the reduction in stress resulting in a 24-percent decrease in weight-flow capacity. The high-output two-stage turbine of point 7 has a weight-flow capacity of 34 lb/(sec)(sq ft); whereas, a conservative two-stage design (not shown) for the same pressure ratio and stress has a weight-flow capacity of 25 lb/(sec)(sq ft). For the design conditions of figure 4(b), a conservative two-stage turbine can outperform a high-output one-stage turbine and can do so with a 40-percent reduction in turbine stress. Since parameter e at point 7 is well below the 44 million lb/sec^3 limit, for the conditions of this chart compressor aerodynamics is not limiting for the current turbine stress limit of 30,000 pounds per square inch. The lowest value of turbine stress for which parameter e has the compressor-limited value of 44 million lb/sec^3 is 49,000 psi.

Turbine-inlet temperature of 2457° R. - Figures 4(c) and (d) differ only in their two-stage designs; the former presents high-output two-stage designs, and the latter, conservative. The one-stage designs of these two charts are identical.

In figure 4(c) at a compressor pressure ratio of 5.0 and turbine blade centrifugal stress of 30,000 pounds per square inch (point 8), a high-output one-stage turbine has a weight-flow capacity of about 20 lb/(sec)(sq ft). A high-output two-stage turbine designed for point 8 has a weight-flow capacity of about 37 lb/(sec)(sq ft). Thus, at a flight Mach number of 2.0 in the stratosphere for both high-output one-stage and two-stage turbines, an increase in turbine-inlet temperature from 2106° to 2457° R can produce an 8-percent increase in turbine-limited specific weight flow. The hub-tip radius ratios of points 7 and 8 are essentially the same for the one-stage turbines; this is also 3275
true for the two two-stage turbine designs compared.

Alternatively, figure 4(c) shows that, at the value of 18.5 lb/(sec)(sq ft) for turbine-limited specific weight flow and 30,000 pounds per square inch for turbine stress that are located at point 7, the 351° R increase in turbine-inlet temperature can raise the compressor pressure ratio obtainable from a high-output one-stage turbine from 5.0 to about 7.0. Also in figure 4(c), point 9 shows that a high-output two-stage turbine has a weight-flow capacity of 36 lb/(sec)(sq ft) at a compressor pressure ratio of 7.2 and turbine blade centrifugal stress of 30,000 pounds per square inch. Both points 8 and 9 lie well below the 44 million lb/sec^3 limit on parameter e. In fact, for the conditions of this chart, the least value of turbine stress at which parameter e becomes limiting is about 42,000 pounds per square inch.

Figure 4(d) is presented to show the effect of lowering the Mach number limit of a two-stage turbine at the same flight conditions and turbine-inlet temperature as used in the construction of figure 4(c). At point 10 the compressor pressure ratio and turbine blade centrifugal stress are the same as for point 9. The turbine-limited specific weight flow at point 10 is about 26 lb/(sec)(sq ft), a 28-percent decrease in weight-flow capacity. Figures 4(c) and (d) show that this decrease in turbine-limited specific weight flow is accompanied by an increase in hub-tip radius ratio from 0.57 to 0.62.

<u>Turbine-inlet temperature of 2808° R</u>. - The chart of figure 4(e) differs from that of figure 4(a) only in the stress curves; lower values of turbine-limited specific weight flow are shown, and compressor pressure ratio is not extended beyond 10.0. For the conditions of this chart, a compressor pressure ratio even as low as one-half the value for minimum specific fuel consumption (see table II), in combination with a high-output one-stage turbine with 0.8 Mach number limits, requires either turbine stress higher than the current limit of 30,000 psi or weight-flow capacity less than 22 lb/(sec)(sq ft). This difficulty is easily eliminated by two-stage turbines of conservative design. Point 11 shows that a conservative two-stage turbine can produce a compressor pressure ratio of 10.0 and a turbine-limited specific weight flow of 27.5 lb/(sec)(sq ft) at 30,000-psi turbine blade centrifugal stress.

Point 12, which corresponds to the compressor pressure ratio and turbine stress of the conservative turbine of point 10, shows that an increase in turbine-inlet temperature from a value of 2457° to 2808° R is accompanied by a 12-percent increase in weight-flow capacity of a conservative two-stage turbine. The hub-tip radius ratios of points 10 and 12 are the same. Despite this 351° R increase in turbine-inlet temperature of point 12 over point 9, the conservative two-stage design of point 12 has 19-percent-lower weight-flow capacity than the high-output two-stage design of point 9. The limiting value of 44 million lb/sec^3 for parameter e is obtained with turbine stresses greater than 37,000 psi, even for high-output turbines.

Critical comment. - For Mach 2.0 engine designs in the stratosphere, compressor aerodynamics does not become a limiting factor until the turbine blade centrifugal stress reaches from 37,000 to 49,000 pounds per square inch, depending on the turbine-inlet temperature. For this flight condition, one-stage turbines must be critical in aerodynamic design. Conservative two-stage turbines have adequate weight-flow and work capacities at current turbine blade centrifugal-stress limits. A conservative two-stage turbine can outperform a high-output one-stage turbine at this flight condition and can do so at a considerably lower turbine stress. Increasing the turbine Mach number limit of a two-stage turbine from 0.6 to 0.8 and maintaining the same turbine-inlet temperature produces more than three times the increase in weight-flow capacity that is obtained by increasing the turbine-inlet temperature by 351° to 2808° R and maintaining the 0.6 Mach number limit. Turbine blade centrifugal stress commences to impose limitations at this flight condition. The compressor pressure ratios obtainable from high-output one-stage turbines at this flight condition are considerably less than those for minimum specific fuel consumption.

Mach 2.8 Designs in Stratosphere

All the Mach 2.8 turbine charts (figs. 4(f) to (i)) show that compressor aerodynamics is no limitation even when the turbine blade centrifugal stress is as high as 60,000 pounds per square inch, because parameter e remains well below 44 million lb/sec^3. This observation is in contrast to that of the Mach 2.0 designs, for figures 4(b) to (e) have shown that parameter e reaches a limit of 44 million lb/sec^3 at turbine blade centrifugal stresses between 37,000 and 49,000 pounds per square inch.

Turbine-inlet temperature of 2003° R. - Because the compressor pressure ratio for minimum specific fuel consumption is so low (2.13) for the 2003° R turbine-inlet temperature and flight Mach number of 2.8, only a small region of this chart (fig. 4(f)) is worthy of consideration. This region of interest, furthermore, is confined to one-stage designs.

Point 13 locates one of the few designs feasible for the conditions specified for this chart. It shows that a high-output one-stage turbine can produce a turbine-limited specific weight flow of 21 lb/(sec)(sq ft) and a compressor pressure ratio of 2.0 at a 30,000-psi turbine blade centrifugal stress. The turbine-inlet temperature of 2003° R is sufficiently low that such a turbine would not require blade cooling.

Turbine-inlet temperature of 2504° R. - The 501° R increase in turbine-inlet temperature from figure 4(f) to 4(g) not only extends the region of feasible one-stage turbine designs, but also admits two-stage designs into consideration. Point 14 is a possible high-output one-stage design within a 30,000-psi stress limit, but the weight-flow capacity of 18 lb/(sec)(sq ft) is rather low. The high-output two-stage turbine design designated by point 15 has a turbine-limited specific weight flow of 30 lb/(sec)(sq ft) at the 30,000-psi stress level and at a compressor pressure ratio only slightly lower than that for minimum specific fuel consumption.

Turbine-inlet temperature of 3004° R. - At the engine temperature ratio of 3.0, a compressor pressure ratio of 5.5 yields minimum specific fuel consumption (figs. 4(h) and (i)). For high-output one-stage turbines having 30,000-psi stress, the turbine-limited specific weight flow is so low at this compressor pressure ratio that a value considerably less than 5.5 should be selected in the interest of keeping turbine size from becoming excessively large. If the compressor pressure ratio is therefore reduced to 3.0, point 16 (fig. 4(h)) shows a turbine design that can produce a turbine-limited specific weight flow of about 18.0 lb/(sec)(sq ft) at 30,000-psi turbine blade centrifugal stress.

An alternative is to raise the turbine stress to, for example, 60,000 pounds per square inch and reduce compressor pressure ratio to 4.0 rather than 3.0. Point 17, designating such a turbine design in figure 4(h), shows that a turbine-limited specific weight flow of 24 lb/(sec)(sq ft) can be produced.

A high-output two-stage turbine, however, can drive a compressor having a pressure ratio of 5.0 with a turbine-limited specific weight flow of 29.5 lb/(sec)(sq ft) at a 30,000-psi blade centrifugal stress (point 18). This weight-flow capacity is sufficiently high to make turbine frontal area unimportant in terms of its effect on nacelle drag. For this reason, the reduced turbine-limited specific weight flow obtainable from a conservative two-stage turbine might be accepted to present the possibility of increased turbine efficiency that might be realized by reason of the more conservative design.

The turbine-exit axial velocity, however, imposes a restriction on the flow capacity of a machine. This fact is illustrated by figure 5 (a plot of eq. (E2)), in which turbine-limited specific weight flow is plotted against compressor pressure ratio for two values of $(V_x/a'_{cr})_7$. In this figure the engine temperature ratio is 3.0, and turbine hub-tip radius ratio is 0.5. A rise in weight-flow capacity associated with a value of $(V_x/a'_{cr})_7$ of 0.7 is about 25 percent above that for a value of 0.5. The curves in figure 5 are independent of the number of turbine stages and show that, for the given engine temperature ratio and a specified flight Mach number, weight-flow capacity comparable with that obtainable in compressor design can be achieved in turbine design only at high values of $(V_x/a'_{cr})_7$. The weight-flow capacity of a given machine cannot have a value above the curves of figure 5 for the same hub-tip radius ratio.

To return to consideration of using a conservative two-stage turbine at point 18, figure 5 shows that, at a compressor pressure ratio of 5.0, a machine with 0.5 exit axial-velocity ratio can produce no more than 27.1 lb/(sec)(sq ft). Furthermore, this value of 27.1 is obtainable only if the turbine hub-tip radius ratio is as low as 0.5. Data from figure 2(a) of reference 3 indicate that a conservative two-stage turbine can produce a compressor pressure ratio of 5.0 and a turbine-limited specific weight flow of 21.5 lb/(sec)(sq ft) at the 30,000-psi turbine stress level. The benefits possible in a two-stage turbine by utilizing the more conservative aerodynamic limits can be achieved at the expense of a 27-percent decrease in weight-flow capacity.

The value of parameter e at point 18 (fig. 4(h)) is only 18.5 million lb/sec^3. Figures 3(a) and (c) show that a compressor matched with this turbine can be conservative in aerodynamic design, having an inlet relative Mach number as low as 0.86 and an equivalent blade tip speed of 810 feet per second. Such a conservative compressor should have capacity for operation at equivalent blade speeds considerably greater than the value during high Mach number flight.

A cursory analysis indicates that two-stage turbines may not be well suited for turbine stator adjustment in off-design operation because of the occurrence of unacceptably high incidence angles for the first row of turbine rotor blades. This combination of conservative compressor operation at the high flight Mach number and the lack of range of efficient two-stage turbine operation with turbine stator adjustment suggests that such an engine may be suitable for operation with constant engine geometry (see ref. 5). For example, a compressor designed for 2.8 flight Mach number would require 39-percent equivalent overspeed capacity at take-off; in the particular case cited, the take-off compressor equivalent blade tip speed would be 1125 feet per second at a compressor blade centrifugal stress of 40,000 pounds per square inch (fig. 2(a)).

Figure 4(i) is presented in order that, by comparison with the one-stage turbines of figure 4(h), the merits of raising the turbine rotor-inlet relative Mach number limit from 0.8 to 1.0 can be ascertained.

For this purpose, point 19 was selected at the same compressor pressure ratio (3.0) and turbine stress (30,000 psi) as point 16. Point 19 shows that the increase in turbine Mach number limit permits the turbine-limited specific weight flow to increase from 18 to 19.5 lb/(sec)(sq ft). This 8-percent increase in flow is achieved solely by the increase in turbine-inlet relative Mach number limit, because both figures 4(h) and (i) were prepared for a value of 0.7 for $(V_x/a'_{cr})_7$, and the hub-tip radius ratios are the same. It should be further recognized that this 8-percent increase in flow is possible only if the turbine efficiency does not decrease as the relative Mach number of the gas entering the turbine rotor increases.

Critical comment. - Compressor aerodynamics imposes no limitations for engine designs at the Mach 2.8 stratosphere flight condition. One-stage turbines must be critical in aerodynamic design; such high-output one-stage turbines have high weight-flow capacity only at turbine blade centrifugal-stress levels of the order of 50,000 to 60,000 pounds per square inch. High turbine-limited specific weight flows are obtainable at 30,000 pounds per square inch turbine stress from high-output two-stage turbines; conservative aerodynamic limits on two-stage turbine designs reduce the weight-flow capacity by 27 percent. This latter combination yields high turbine efficiency and high turbine weight.

High-output two-stage turbines have sufficient work and weight-flow capacities at such low equivalent blade tip speeds that compressor operation can be very conservative at the high flight Mach number design point. Such a compressor may have great latitude in compressor equivalent overspeed, a desirable off-design characteristic. An increase in turbine-inlet relative Mach number limit from 0.8 to 1.0 yields an 8-percent increase in turbine-limited specific weight flow under the assumption that turbine efficiency remains constant.

CONCLUSIONS

From a design-point analysis relating engine and compressor and turbine design parameters of one-spool turbojet engines, the following conclusions are drawn:

1. Compressor aerodynamics currently imposes a severe limitation on static sea-level engine designs, but does not become limiting for Mach 2.0 designs in the stratosphere until turbine blade centrifugal

stresses become 37,000 to 49,000 pounds per square inch, depending upon the turbine-inlet temperature. For design-point operation of turbojet engines at 2.8 flight Mach number, compressor aerodynamics is not limiting.

2. Turbine aerodynamics is critical for one-stage turbines of engines designed for static sea-level operation as well as for operation at flight Mach numbers of 2.0 and 2.8 in the stratosphere. Conservative two-stage turbines have adequate weight-flow and work capacities within
3275 the current limit on centrifugal stress of 30,000 pounds per square inch for both sea-level static operation and Mach 2.0 flight in the stratosphere. The benefits obtainable from using conservative aerodynamic limits in two-stage turbines for Mach 2.8 flight in the stratosphere are at the expense of a 27-percent decrease in weight-flow capacity.

3. In the stratosphere for both one- and two-stage turbines, turbine blade centrifugal stress becomes a limiting factor for Mach 2.0 engine designs and is a primary constraint for Mach 2.8 engine designs.

4. During static operation at sea level, conservative two-stage turbines are capable of driving compressors of high pressure ratio and high equivalent specific air flow without exceeding either limits on compressor aerodynamics or a centrifugal stress in the turbine rotor blades of 30,000 pounds per square inch.

5. For engines designed either for static operation at sea level or for a flight Mach number of 2.0 in the stratosphere, the compressor pressure ratios that can be produced by one-stage turbines limited by 0.8 turbine-inlet relative Mach number are considerably less than that for minimum specific fuel consumption of afterburning engines if turbine stress is not to become excessive nor weight-flow capacity too low.

6. For flight Mach numbers of 2.8 and higher, two-stage turbines appear capable of driving compressors of suitable compressor pressure ratio at such low equivalent blade tip speeds that the compressor operation can be very conservative during the high flight Mach number operation. Such a conservative compressor might well possess the capacity for operation at equivalent blade tip speeds higher than the value during high Mach number flight, a useful characteristic for providing good low flight Mach number operation.

7. Gains in turbine-limited specific weight flow of the order of 8 percent are possible in one-stage turbines designed for 2.8 flight Mach number, if the limit on turbine rotor-inlet relative Mach number is increased from 0.8 to 1.0, provided that turbine efficiency does not deteriorate in the process.

Lewis Flight Propulsion Laboratory
National Advisory Committee for Aeronautics
Cleveland, Ohio, June 30, 1954

APPENDIX A

SYMBOLS

The following symbols are used in this report:

A annular area, sq ft

A_A afterburner frontal area, sq ft 3275

A_C compressor frontal area, sq ft

A_T turbine frontal area, sq ft

a sonic velocity, $\sqrt{\gamma gRT}$ or $\sqrt{kgRT}$, ft/sec

a'_{cr} critical velocity relative to stator, $\sqrt{\frac{2k}{k+1} gRT'}$, ft/sec

b fraction of weight flow bled from compressor

C_n exhaust-nozzle velocity coefficient

e parameter used in relating compressors and turbines, $\frac{wU_t^2}{A_t \delta_1' \sqrt{\theta_1'}}$, lb/sec³

f fuel-air ratio, lb fuel/lb air

g gravitational constant, 32.2 ft/sec²

h specific enthalpy, Btu/lb

J mechanical equivalent of heat, 778.2 ft-lb/Btu

k ratio of specific heats for hot gas, 4/3

M Mach number relative to rotating blades

p absolute pressure, lb/sq ft

R gas constant, 53.4 ft-lb/(lb)(°R)

r radius, ft

s specific entropy, Btu/(lb)(°R)

sfc specific fuel consumption, lb fuel/(lb thrust)(hr)

T temperature, °R

U blade velocity, ft/sec

3275 V absolute velocity of gas, ft/sec

v square of compressor-inlet Mach number, $(V/a)^2$

W relative velocity of gas, ft/sec

w weight flow, lb/sec

Γ density of blade metal, lb/cu ft

γ ratio of specific heats for air, 1.40

δ ratio of pressure to NACA standard sea-level pressure, p/2116.216 (ref. 6)

η adiabatic efficiency

θ ratio of temperature to NACA standard sea-level temperature, T/518.688 (ref. 6)

ρ density of gas, lb/cu ft

σ blade centrifugal stress at hub radius, psi

ψ stress-correction factor for tapered blades

ω angular velocity, radians/sec

Subscripts:

C compressor

h hub

m mean

max maximized

s isentropic

T turbine

t tip

u tangential component

x axial component

0 free stream

1 compressor inlet

2 compressor exit 3275

3 turbine inlet

4 exit from first turbine stator

5 exit from first turbine rotor

6 exit from second turbine stator

7 turbine exit

8 afterburner inlet

9 afterburner exit

10 exhaust-nozzle exit

Superscripts:

' stagnation state relative to stator

APPENDIX B

ASSUMPTIONS AND CONSTANTS

Assumptions

The following assumptions were used in the turbine portion of the analysis:

(1) Simplified radial equilibrium

(2) Free-vortex velocity distribution

(3) At the mean radius, $(\rho V_x)_m A$ equal to integrated value of weight flow over blade height:

$$w = (\rho V_x)_{4,m} A_4 = (\rho V_x)_{5,m} A_5$$

$$w = (\rho V_x)_{6,m} A_6 = (\rho V_x)_{7,m} A_7$$

(4) No radial variation in stagnation state relative to stator

(5) Hub and mean radii constant in value from entrance to exit of each rotor blade row

The following turbine design parameters are postulated to limit the aerodynamic design of the turbines analyzed herein:

(a) Exit critical axial-velocity ratio $(V_x/a'_{cr})_7$ (see table I)

(b) Mach number at hub radius and entrance of blade row (see table I)

(c) Zero change in magnitude of relative velocity across rotor at hub radius (applies to one-stage turbines and second stage of two-stage turbines)

(d) 120° Rotor turning angle at hub radius (applies only to first stage of two-stage turbines)

(e) Tangential component of velocity at turbine exit (see following list of constants)

Constants

The constants used in the analysis are as follows:

Compressor adiabatic efficiency, η_C	0.85
Combustor stagnation pressure ratio, p_3'/p_2'	0.95
Exit-to-inlet annular area ratio for one-stage turbines, A_7/A_6	1.0
Exit-to-inlet isentropic annular area ratio for each stage of two-stage turbines, $(A_7/A_6)_s$, $(A_5/A_4)_s$	1.0
Density of compressor and turbine blade metals, Γ_C, Γ_T, lb/cu ft	500
Stress-correction factor for tapered blades, ψ_C, ψ_T	0.7
Turbine-exit whirl (fraction of stagnation enthalpy drop of one-stage turbines $\dfrac{V_{u,7,h}^2/2gJ}{h_3' - h_7'}$ or fraction of second-stage enthalpy drop of two-stage turbines $\dfrac{V_{u,7,h}^2/2gJ}{h_5' - h_7'}$)	0.02
Turbine adiabatic efficiency, η_T	0.85

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APPENDIX C

CALCULATION OF PARAMETER e FOR COMPRESSOR

For a compressor without inlet guide vanes, the rotor-inlet relative Mach number at the tip is

$$M_1 = \sqrt{\left(\frac{U_{t,C}}{a_1}\right)^2 + \left(\frac{V}{a}\right)_1^2} \tag{C1}$$

from which

$$\frac{U_{t,C}}{a_1} = \sqrt{M_1^2 - \left(\frac{V}{a}\right)_1^2} \tag{C2}$$

Compressor equivalent blade tip speed is

$$\frac{U_{t,C}}{\sqrt{\theta_1'}} = \frac{U_{t,C}}{a_1} \sqrt{\gamma g R 518.7} \sqrt{\left(\frac{T}{T'}\right)_1} \tag{C3}$$

which, with equation (C2), is

$$\frac{U_{t,C}}{\sqrt{\theta_1'}} = \sqrt{518.7} \sqrt{\gamma g R} \sqrt{M_1^2 - \left(\frac{V}{a}\right)_1^2} \sqrt{\left(\frac{T}{T'}\right)_1} \tag{C4}$$

By use of the energy equation

$$\left(\frac{T'}{T}\right)_1 = 1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2 \tag{C5}$$

equation (C4) becomes

$$\frac{U_{t,C}}{\sqrt{\theta_1'}} = \sqrt{518.7} \sqrt{\gamma g R} \sqrt{\frac{M_1^2 - \left(\frac{V}{a}\right)_1^2}{1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2}} \tag{C6}$$

Equivalent specific air flow is

$$\frac{w_C\sqrt{\theta_1'}}{A_C\delta_1'} = \frac{2116}{\sqrt{518.7}}\sqrt{\frac{\gamma g}{R}}\left(\frac{\rho V}{\rho' a'}\right)_1\left[1 - \left(\frac{r_h}{r_t}\right)_C^2\right] \tag{C7}$$

where

$$\left(\frac{\rho'}{\rho}\right)_1 = \left(\frac{T'}{T}\right)_1^{\frac{1}{\gamma-1}} \tag{C8}$$ 3275

and

$$\left(\frac{V}{a'}\right)_1 = \left(\frac{V}{a}\right)_1\sqrt{\left(\frac{T}{T'}\right)_1} \tag{C9}$$

It was assumed that no radial variation occurred in the flow at the compressor inlet. Substitution of equations (C5), (C8), and (C9) into equation (C7) yields for equivalent air flow

$$\frac{w_C\sqrt{\theta_1'}}{A_C\delta_1'} = \frac{2116}{\sqrt{518.7}}\sqrt{\frac{\gamma g}{R}}\frac{\left(\frac{V}{a}\right)_1}{\left[1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2\right]^{\frac{\gamma+1}{2(\gamma-1)}}}\left[1 - \left(\frac{r_h}{r_t}\right)_C^2\right] \tag{C10}$$

Parameter e with respect to the compressor is the product of equation (C10) and the square of equation (C6):

$$e = \frac{w_C U_{t,C}^2}{A_C\delta_1'\sqrt{\theta_1'}} = 2116\sqrt{518.7}\sqrt{R(\gamma g)^3}\frac{\left(\frac{V}{a}\right)_1\left[M_1^2 - \left(\frac{V}{a}\right)_1^2\right]}{\left[1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2\right]^{\frac{3\gamma-1}{2(\gamma-1)}}}\left[1 - \left(\frac{r_h}{r_t}\right)_C^2\right] \tag{C11}$$

APPENDIX D

CONDITION AT COMPRESSOR INLET FOR MAXIMIZED PARAMETER e

Differentiation of parameter e (eq. (C11)) with respect to compressor-inlet Mach number $(V/a)_1$ yields

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$$\frac{d\left(\frac{w_C U_{t,C}^2}{A_C \delta_1' \sqrt{\theta_1'}}\right)}{d\left(\frac{V}{a}\right)_1}\left(\frac{w_C U_{t,C}^2}{A_C \delta_1' \sqrt{\theta_1'}}\right)^{-1} = \left(\frac{V}{a}\right)_1^{-1} + \frac{2\left[M_1 \frac{dM_1}{d\left(\frac{V}{a}\right)_1} - \left(\frac{V}{a}\right)_1\right]}{M_1^2 - \left(\frac{V}{a}\right)_1^2} - \frac{3\gamma-1}{2}\left(\frac{V}{a}\right)_1\left[1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2\right]^{-1} \tag{D1}$$

The value of $(V/a)_1$ can now be found which, for any given compressor-inlet relative Mach number at the compressor tip M_1, results in the maximum value of parameter e. A value of M_1 can be arbitrarily selected, because both M_1 and $(V/a)_1$ are independent variables in equation (C2). Then,

$$\frac{d\left(\frac{w_C U_{t,C}^2}{A_C \delta_1' \sqrt{\theta_1'}}\right)}{d\left(\frac{V}{a}\right)_1} = 0$$

and

$$\frac{dM_1}{d\left(\frac{V}{a}\right)_1} = 0$$

Equation (D1) then becomes

$$1 - \frac{2\left(\frac{V}{a}\right)_1^2}{M_1^2 - \left(\frac{V}{a}\right)_1^2} - \frac{3\gamma-1}{2} \frac{\left(\frac{V}{a}\right)_1^2}{1 + \frac{\gamma-1}{2}\left(\frac{V}{a}\right)_1^2} = 0 \tag{D2}$$

For simplicity, let 3275

$$v = \left(\frac{V}{a}\right)_1^2$$

Equation (D2) is then

$$v^2 - (\gamma M_1^2 + 3)v + M_1^2 = 0 \tag{D3}$$

Solution of the quadratic equation (D3) is

$$v = \frac{1}{2}\left[\gamma M_1^2 + 3 \pm \sqrt{(\gamma M_1^2 + 3)^2 - 4M_1^2}\right] \tag{D4}$$

Examination of equation (D4) shows that, if the positive root is taken, imaginary values of $U_{t,c}/a_1$ will be obtained from equation (C2). Thus,

$$\left(\frac{V}{a}\right)_1 = \sqrt{\frac{1}{2}\left[\gamma M_1^2 + 3 - \sqrt{(\gamma M_1^2 + 3)^2 - 4M_1^2}\right]} \tag{D5}$$

for maximized parameter e.

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APPENDIX E

CALCULATION OF PARAMETER e FOR TURBINE

The centrifugal stress at the turbine rotor blade hub radius can be expressed as

$$\sigma_T = \frac{\Gamma_T U_{t,T}^2 \psi_T}{2g(144)} \left[1 - \left(\frac{r_h}{r_t}\right)_T^2\right] \tag{E1}$$

by equation (8) of reference 4. From the continuity relation, the turbine-limited specific weight flow is

$$\frac{w_T\sqrt{\theta_1'}}{A_T\delta_1'} = \frac{2116}{\sqrt{518.7}} \sqrt{\frac{2kg}{(k+1)R}} \left(\frac{\rho V_x}{\rho' a_{cr}'}\right)_{7,m} \left[1 - \left(\frac{r_h}{r_t}\right)_T^2\right] \frac{p_7'/p_1'}{\sqrt{T_7'/T_1'}} \tag{E2}$$

Substitution of equation (E1) into (E2) yields parameter e:

$$\frac{w_T\sqrt{\theta_1'}}{A_T\delta_1'} \left(\frac{U_{t,T}}{\sqrt{\theta_1'}}\right)^2 = \frac{2116}{\sqrt{518.7}} 288 \sqrt{\frac{2kg^3}{(k+1)R}} \left(\frac{\rho V_x}{\rho' a_{cr}'}\right)_{7,m} \left(\frac{\sigma_T}{\theta_1' \Gamma_T \psi_T}\right) \frac{p_7'/p_1'}{\sqrt{T_7'/T_1'}} \tag{E3}$$

The ratio T_7'/T_1' is expressible as

$$\frac{T_7'}{T_1'} = \frac{T_3'}{T_1'} \frac{T_7'}{T_3'} \tag{E4}$$

and p_7'/p_1' as

$$\frac{p_7'}{p_1'} = \frac{p_7'}{p_3'} \frac{p_3'}{p_2'} \frac{p_2'}{p_1'} \tag{E5}$$

The turbine stagnation temperature ratio is calculated from the work equation:

$$w_T(h_3' - h_7') = w_C(h_2' - h_1') \tag{E6}$$

Neglecting fuel-air ratio and compressor bleed this gives

$$\frac{k}{k-1}\frac{R}{J}T_3'\left(1-\frac{T_7'}{T_3'}\right)=\frac{\gamma}{\gamma-1}\frac{R}{J}\frac{T_1'}{\eta_C}\left[\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

from which the turbine stagnation temperature ratio is 3275

$$\frac{T_7'}{T_3'}=1-\frac{\gamma}{\gamma-1}\frac{k-1}{k}\frac{\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}}-1}{\eta_C\frac{T_3'}{T_1'}} \tag{E7}$$

Also, from equation (E6),

$$\eta_T\frac{k}{k-1}\frac{R}{J}T_3'\left[1-\left(\frac{p_7'}{p_3'}\right)^{\frac{k-1}{k}}\right]=\frac{\gamma}{\gamma-1}\frac{R}{J}\frac{T_1'}{\eta_C}\left[\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}}-1\right]$$

from which the turbine stagnation pressure ratio is

$$\frac{p_7'}{p_3'}=\left[1-\frac{\gamma}{\gamma-1}\frac{k-1}{k}\frac{\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}}-1}{\eta_T\eta_C\frac{T_3'}{T_1'}}\right]^{\frac{k}{k-1}} \tag{E8}$$

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Combination of equations (E3) to (E8) yields

$$\frac{w_T U_{t,T}^2}{A_T \delta_1' \sqrt{\theta_1'}} = \frac{2116}{\sqrt{518.7}}\, 288 \sqrt{\frac{2kg^3}{(k+1)R}} \left(\frac{\rho V_x}{\rho' a_{cr}'}\right)_{7,m} \frac{\sigma_T}{\theta_1' \Gamma_T \psi_T} \times$$

$$\frac{\left[1 - \frac{\gamma}{\gamma-1}\,\frac{k-1}{k}\,\frac{\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_T \eta_C \frac{T_3'}{T_1'}}\right]^{\frac{k}{k-1}} \frac{p_2'}{p_1'}\,\frac{p_3'}{p_2'}}{\left[1 - \frac{\gamma}{\gamma-1}\,\frac{k-1}{k}\,\frac{\left(\frac{p_2'}{p_1'}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\eta_C \frac{T_3'}{T_1'}}\right]^{\frac{1}{2}} \sqrt{\frac{T_3'}{T_1'}}} \qquad \text{(E9)}$$

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5. English, Robert E., and Cavicchi, Richard H.: Analysis of Off-Design Operation of High Mach Number Supersonic Turbojet Engines. NACA RM E53J01, 1954.

6. International Civil Aviation Organization and Langley Aeronautical Laboratory: Manual of the ICAO Standard Atmosphere - Calculations by the NACA. NACA TN 3182, 1954. (Supersedes and extends NACA Rep. 218.)

TABLE I. - ENGINE PARAMETERS AND TURBINE AERODYNAMIC PARAMETERS PRESENTED IN TURBINE CHARTS

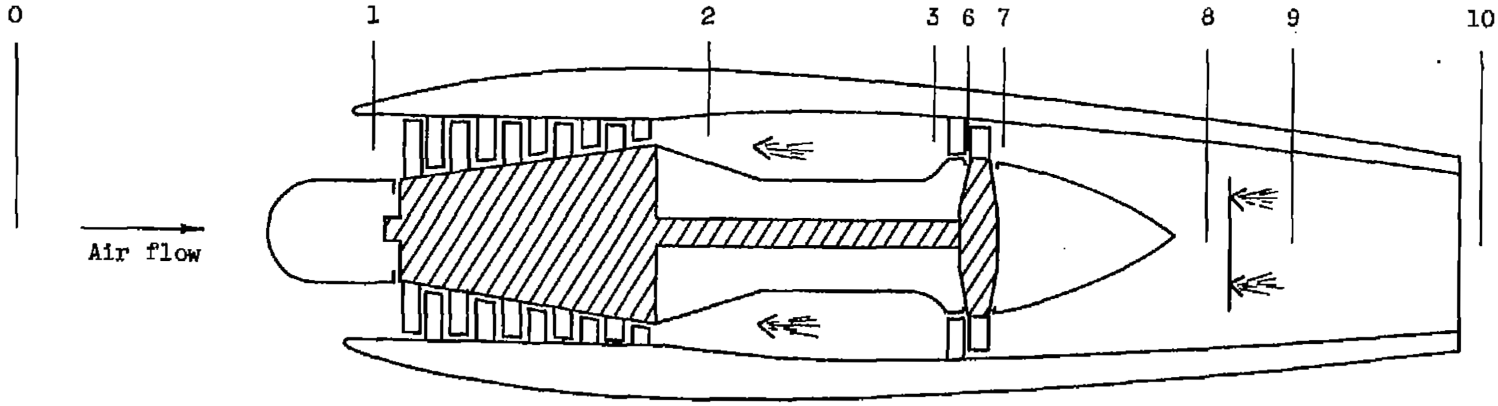
Chart, fig. 4	M_0	Altitude	$\frac{T_3'}{T_1'}$	T_3', °R	One-stage turbines			Two-stage turbines			
					$\left(\frac{W}{a}\right)_{6,h}$	$\left(\frac{V_x}{a_{cr}'}\right)_7$	$\left(\frac{\rho V_x}{\rho' a_{cr}'}\right)_{7,m}$	$\left(\frac{W}{a}\right)_{4,h}$	$\left(\frac{W}{a}\right)_{6,h}$	$\left(\frac{V_x}{a_{cr}'}\right)_7$	$\left(\frac{\rho V_x}{\rho' a_{cr}'}\right)_{7,m}$
(a)	0	Sea level	4.0	2075	$\leq$ 0.8	0.7	0.562	0.6	$\leq$ 0.6	0.5	0.448
(a)	1.28	Stratosphere	4.0	2075	$\leq$.8	.7	.562	.6	$\leq$.6	.5	.448
(b)	2.0	Stratosphere	3.0	2106	$\leq$.8	.7	.562	.8	$\leq$.8	.7	.562
(c)	2.0	Stratosphere	3.5	2457	$\leq$.8	.7	.562	.8	$\leq$.8	.7	.562
(d)	2.0	Stratosphere	3.5	2457	$\leq$.8	.7	.562	.6	$\leq$.6	.5	.448
(e)	2.0	Stratosphere	4.0	2808	$\leq$.8	.7	.562	.6	$\leq$.6	.5	.448
(f)	2.8	Stratosphere	2.0	2003	$\leq$.8	.7	.562	.8	$\leq$.8	.7	.562
(g)	2.8	Stratosphere	2.5	2504	$\leq$.8	.7	.562	.8	$\leq$.8	.7	.562
(h)	2.8	Stratosphere	3.0	3004	$\leq$.8	.7	.562	.8	$\leq$.8	.7	.562
(i)	2.8	Stratosphere	3.0	3004	$\leq$ 1.0	.7	.562				

TABLE II. - TABULATION OF COMPRESSOR PRESSURE RATIO

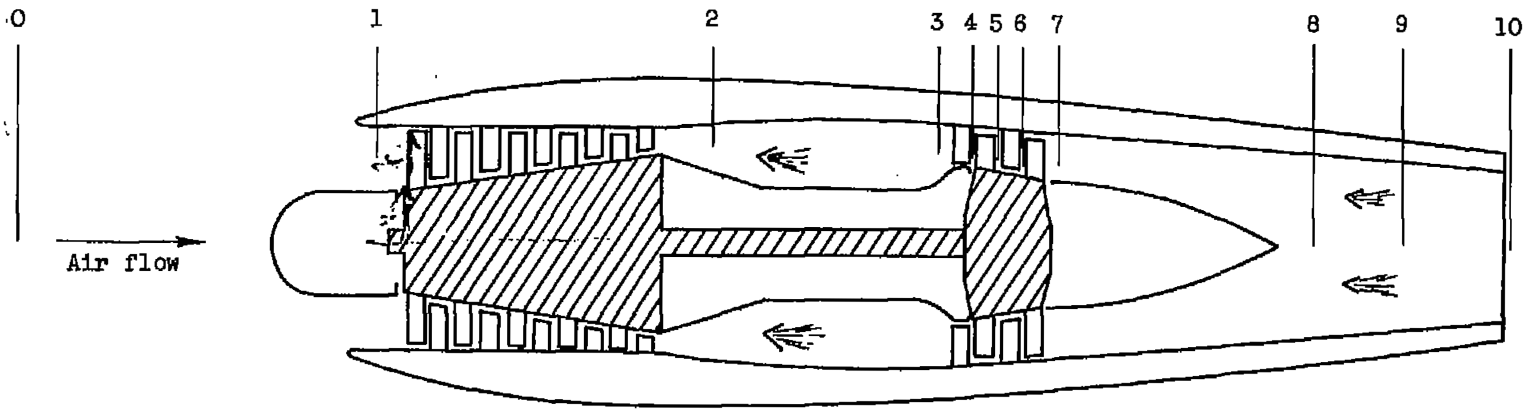
WITH ENGINE TEMPERATURE RATIO FOR MINIMUM SPECIFIC

FUEL CONSUMPTION OF AFTERBURNING ENGINES

Engine temp. ratio, T_3'/T_1'	Compressor pressure ratio p_2'/p_1' for minimum sfc, constant afterburner-exit temperature
2.0	2.1
2.5	3.5
3.0	5.5
3.5	7.9
4.0	11.6



(a) One-stage turbine.



(b) Two-stage turbine.

Figure 1. - Cross section of turbojet engine showing location of numerical stations.

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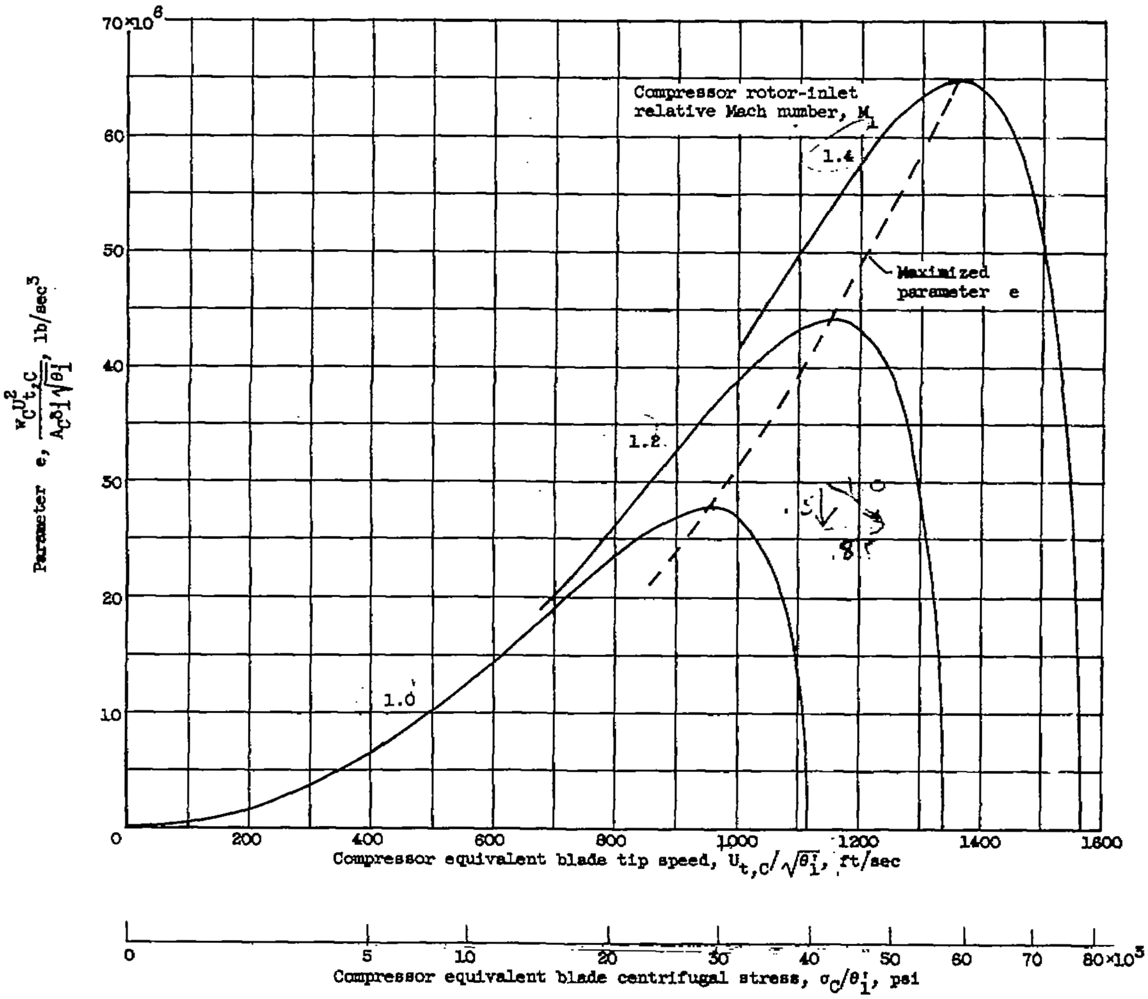
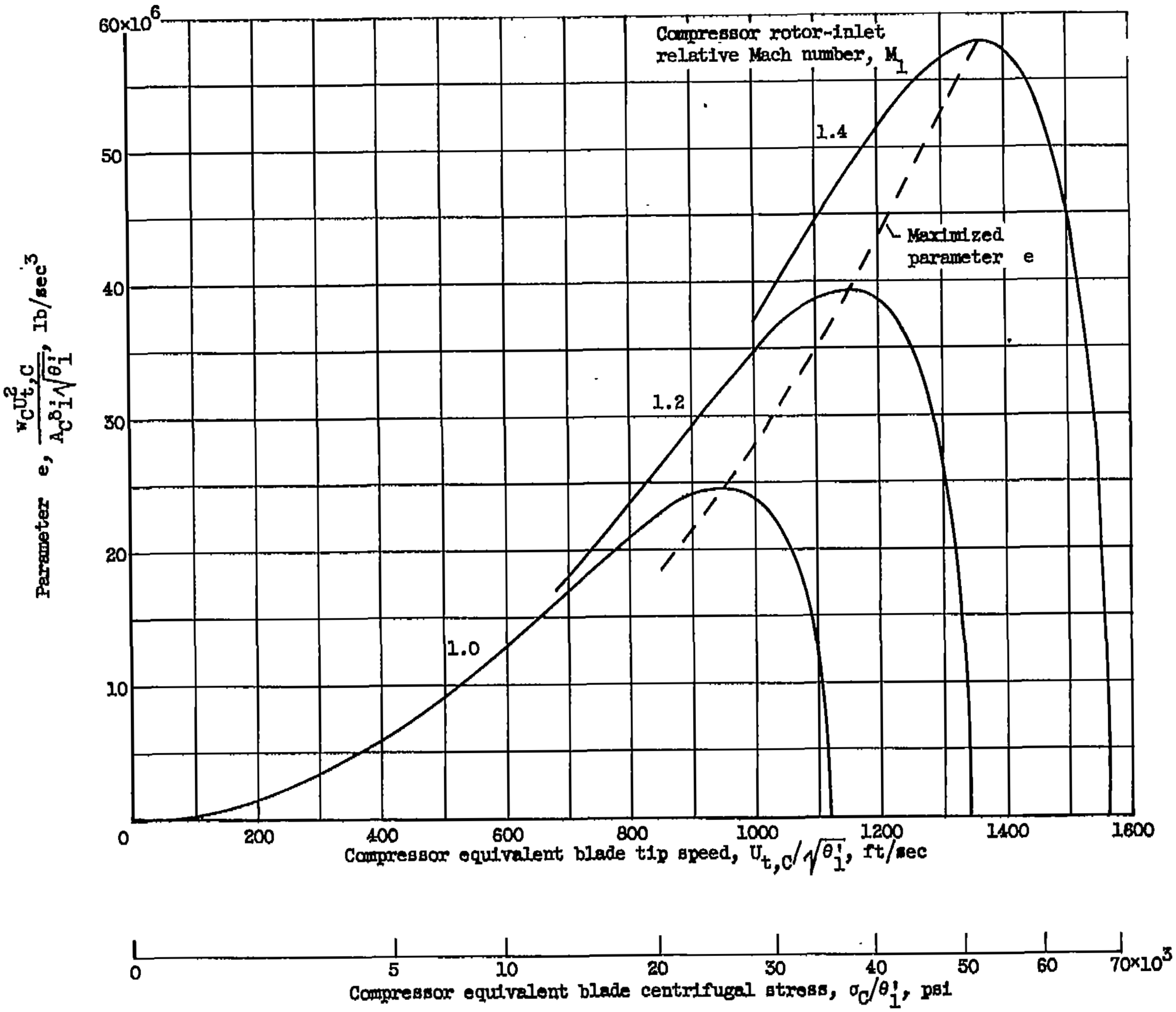


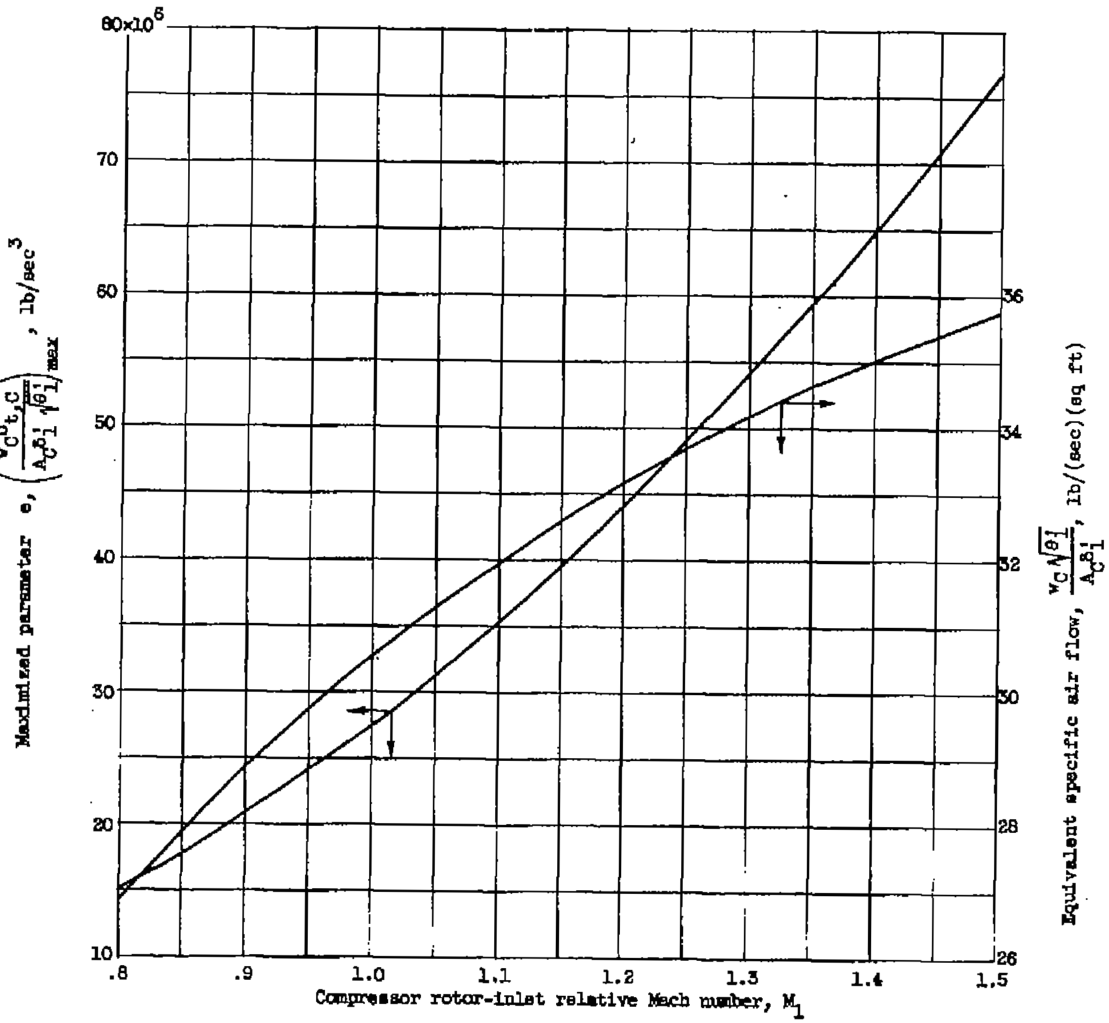
(a) Compressor hub-tip radius ratio, 0.4.

Figure 2. - Compressor charts.



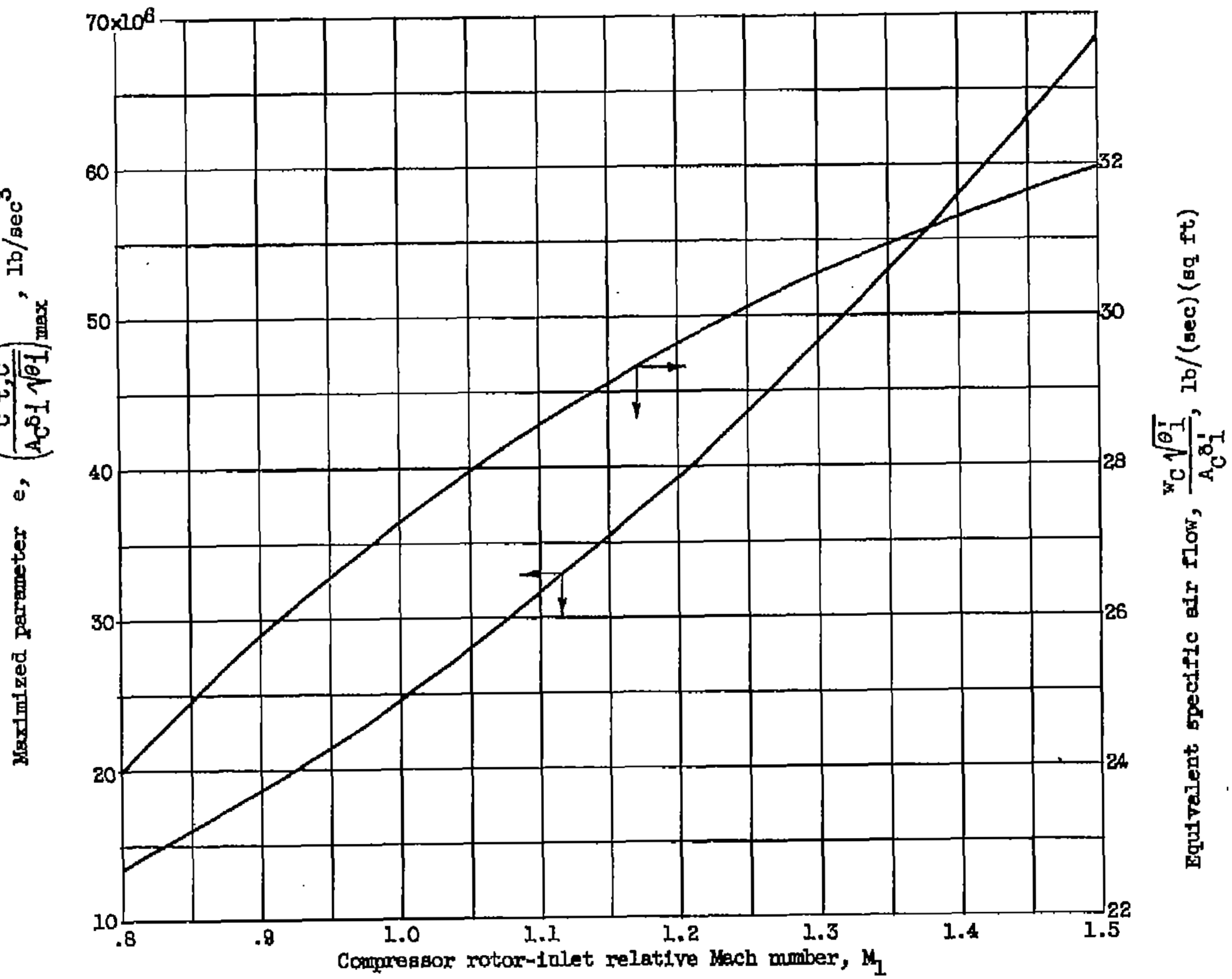
(b) Compressor hub-tip radius ratio, 0.5.

Figure 2. - Concluded. Compressor charts.



(a) Variation of maximized parameter e and equivalent specific air flow with compressor rotor-inlet relative Mach number. Compressor hub-tip radius ratio, 0.4.

Figure 3. - Compressor charts for maximized parameter e.



(b) Variation of maximized parameter e and equivalent specific air flow with compressor rotor-inlet relative Mach number. Compressor hub-tip radius ratio, 0.5.

Figure 3. - Continued. Compressor charts for maximized parameter e.

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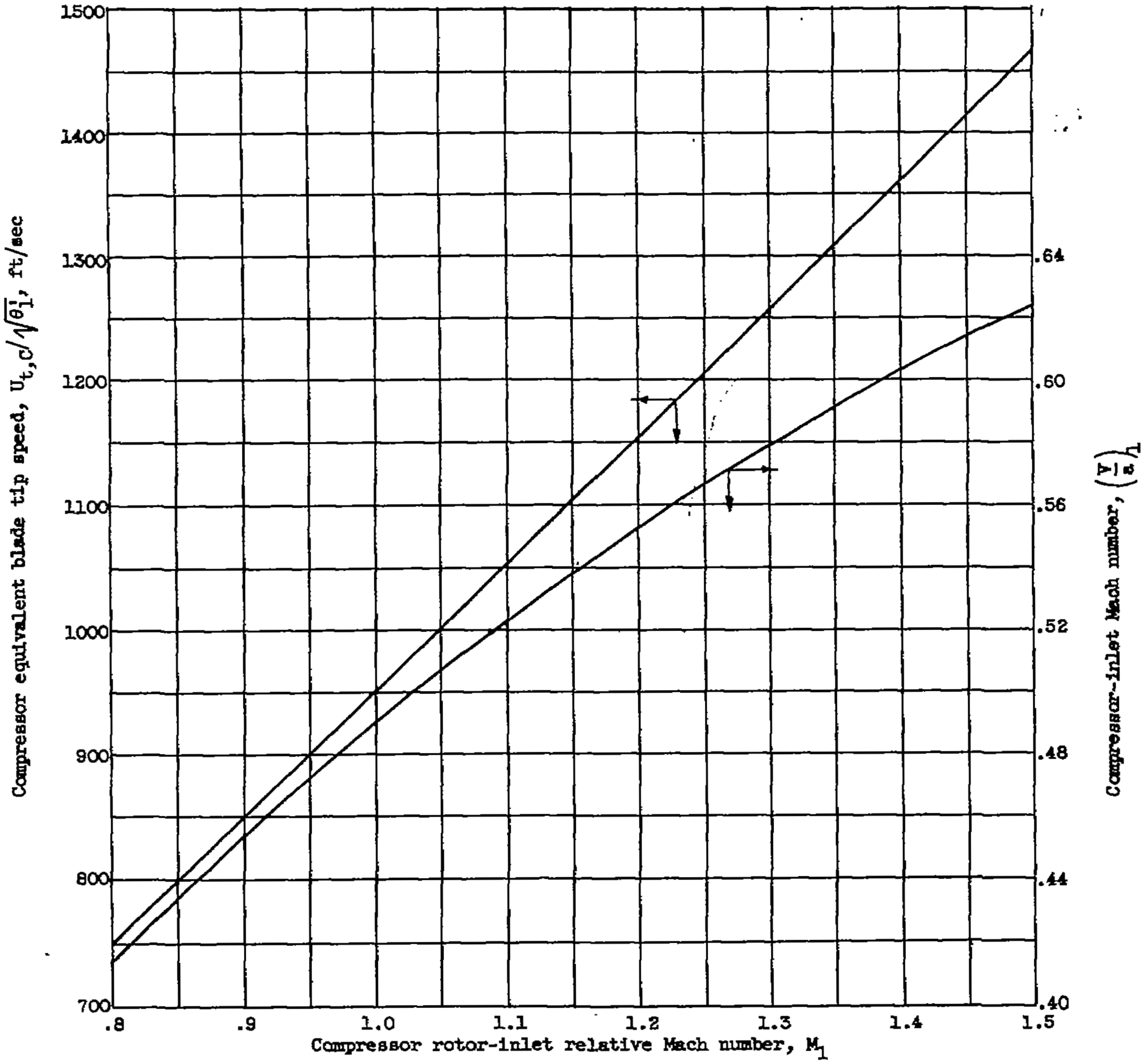
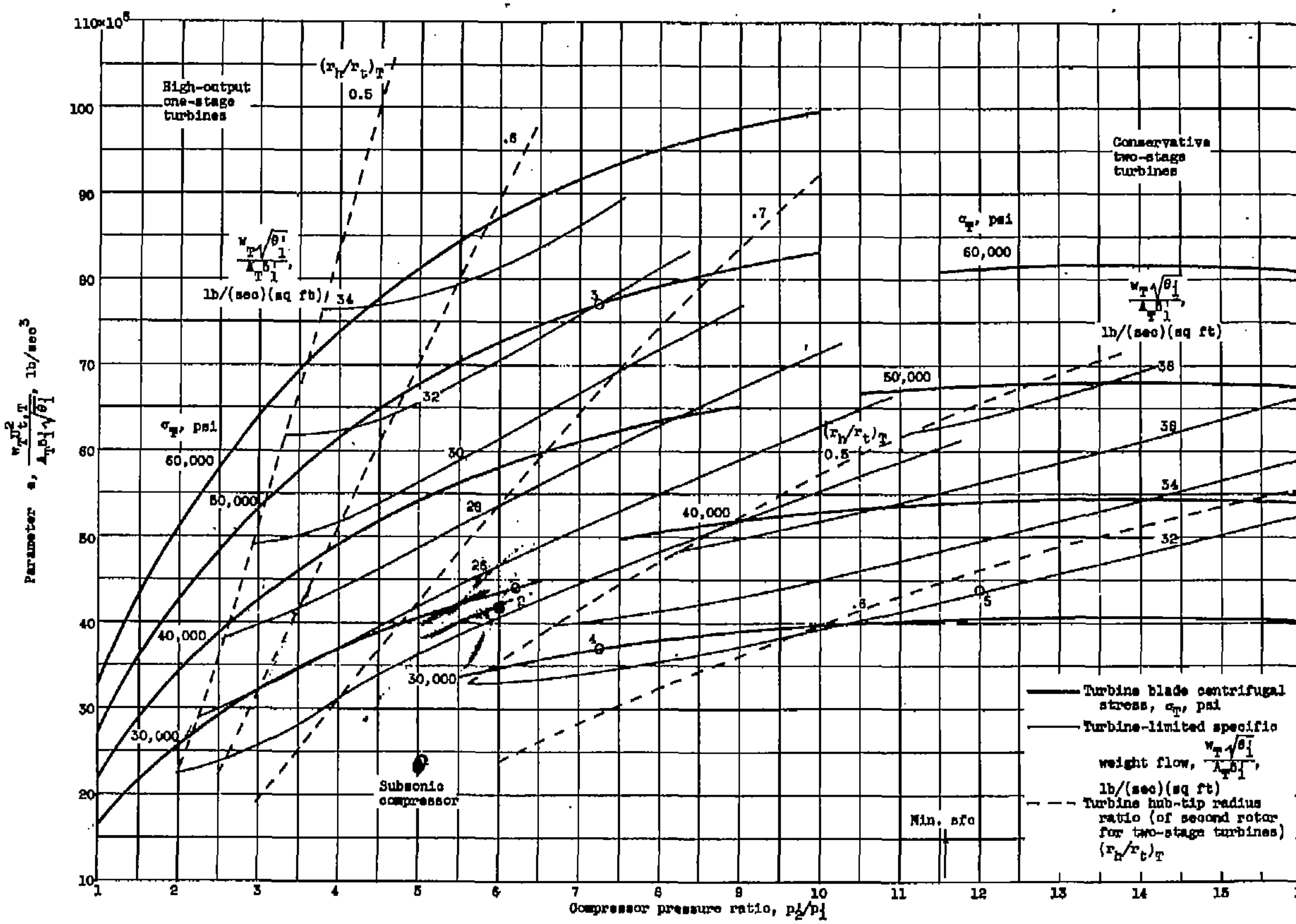


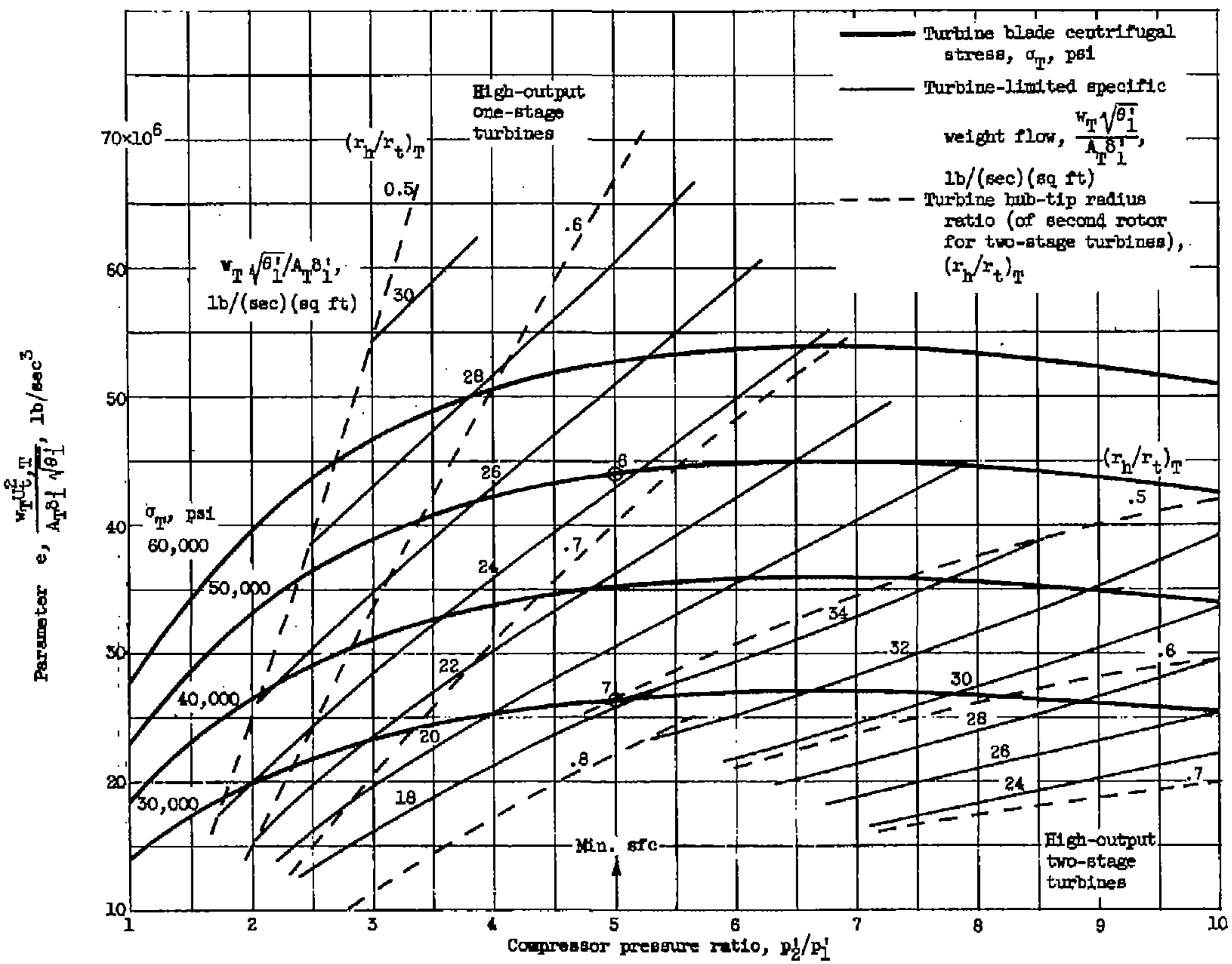
(c) Variation of compressor equivalent blade tip speed and compressor-inlet Mach number with compressor rotor-inlet relative Mach number.

Figure 3. - Concluded. Compressor charts for maximized parameter e.



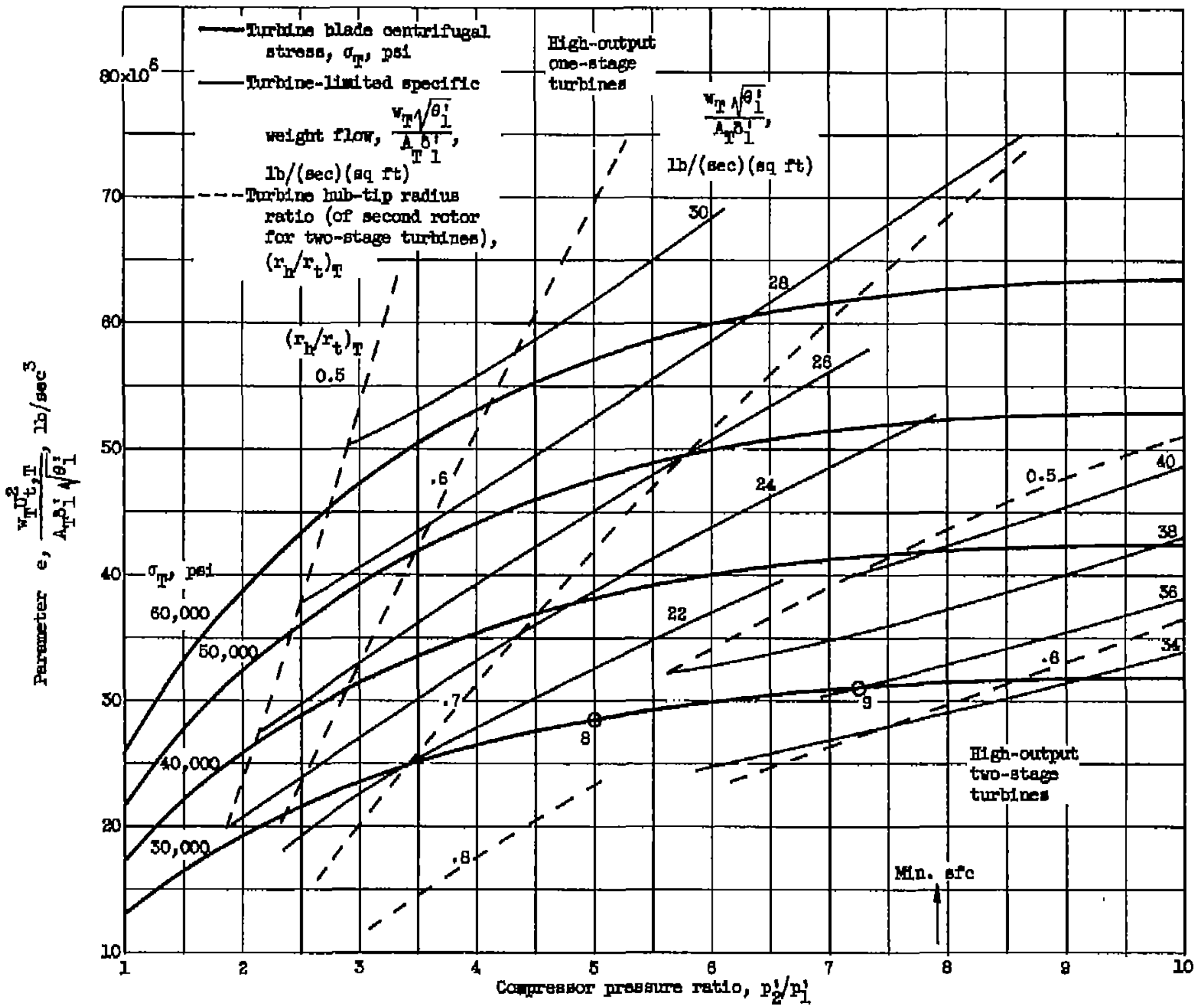
(a) Flight Mach number of 0 at sea level or 1.28 in stratosphere; engine temperature ratio, 4.0; maximum turbine relative Mach number, 0.8 for one-stage turbines, 0.6 for two-stage turbines.

Figure 4. - Turbine charts.



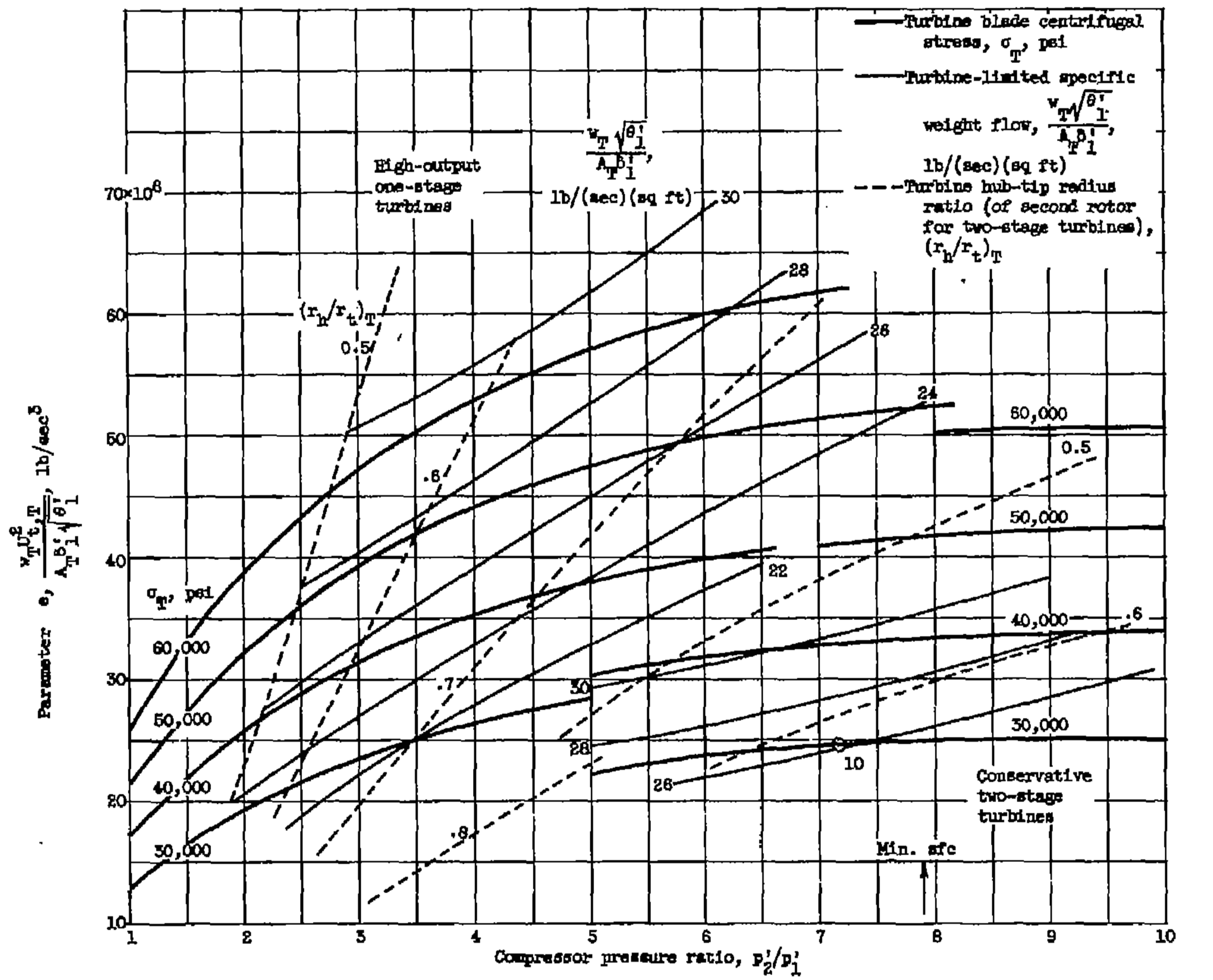
(b) Flight Mach number of 2.0 in stratosphere; engine temperature ratio, 3.0; maximum turbine relative Mach number, 0.8.

Figure 4. - Continued. Turbine charts.



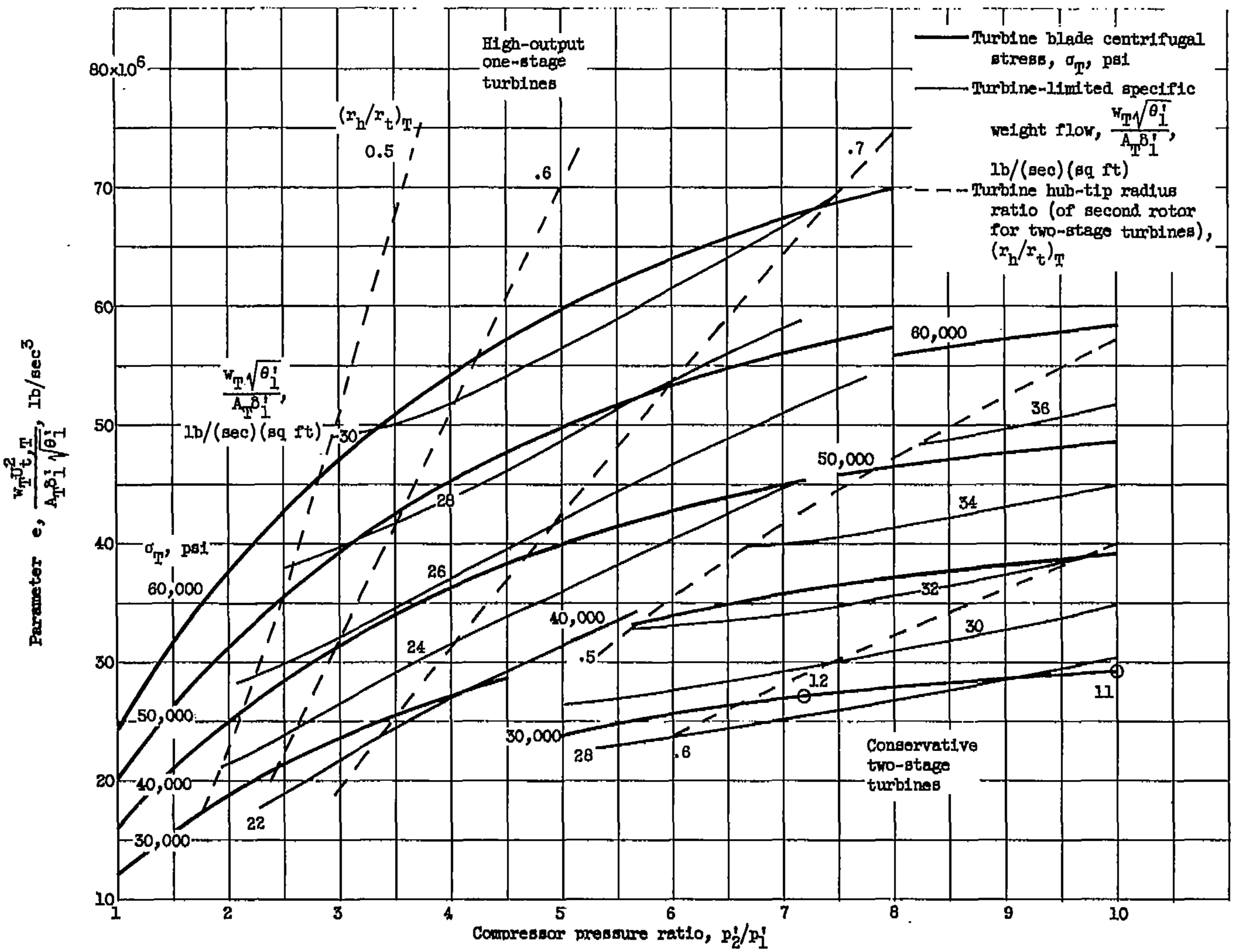
(c) Flight Mach number of 2.0 in stratosphere; engine temperature ratio, 3.5; maximum turbine relative Mach number, 0.8.

Figure 4. - Continued. Turbine charts.



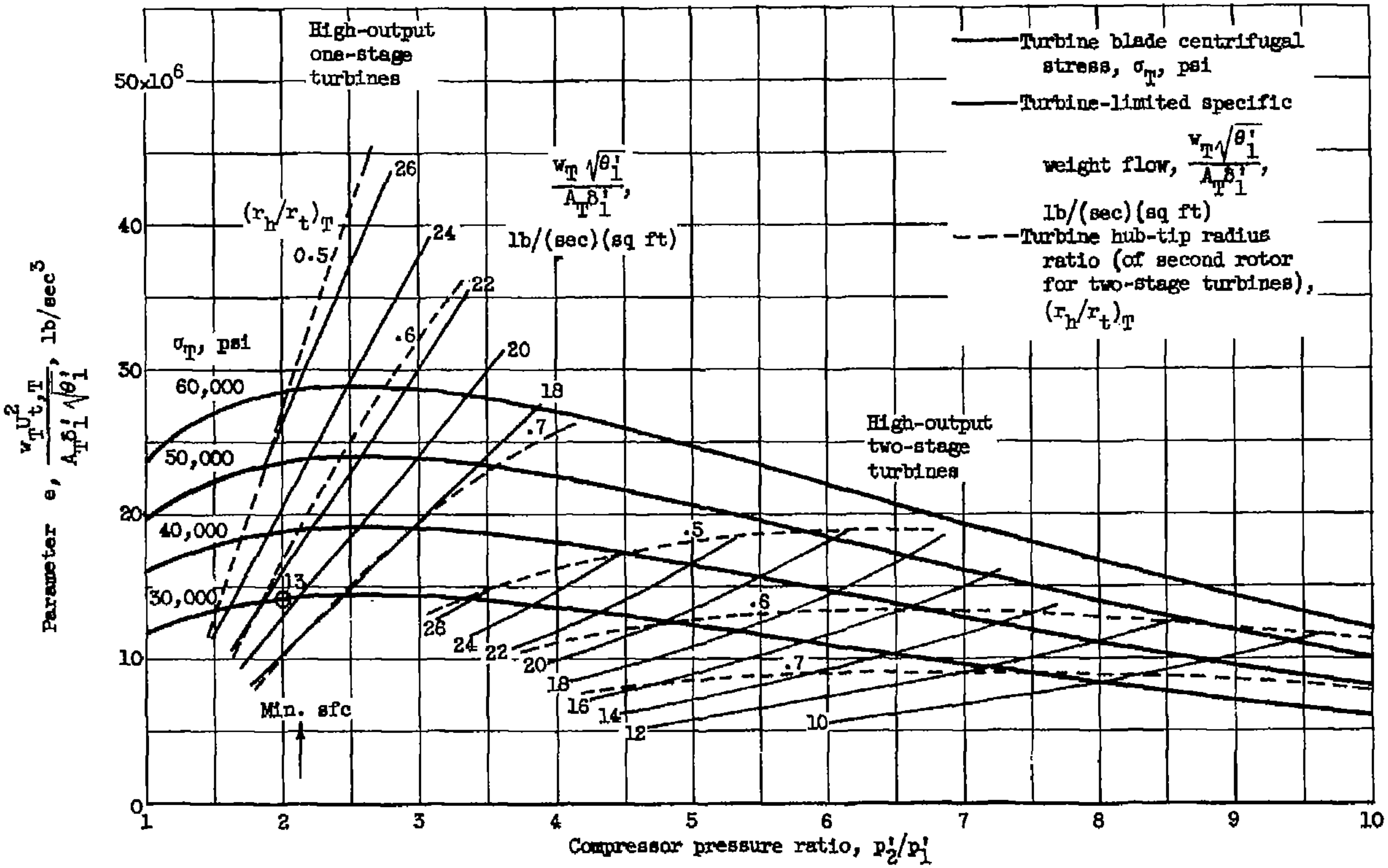
(d) Flight Mach number of 2.0 in stratosphere; engine temperature ratio, 3.5; maximum turbine relative Mach number, 0.8 for one-stage turbines, 0.6 for two-stage turbines.

Figure 4. - Continued. Turbine charts.



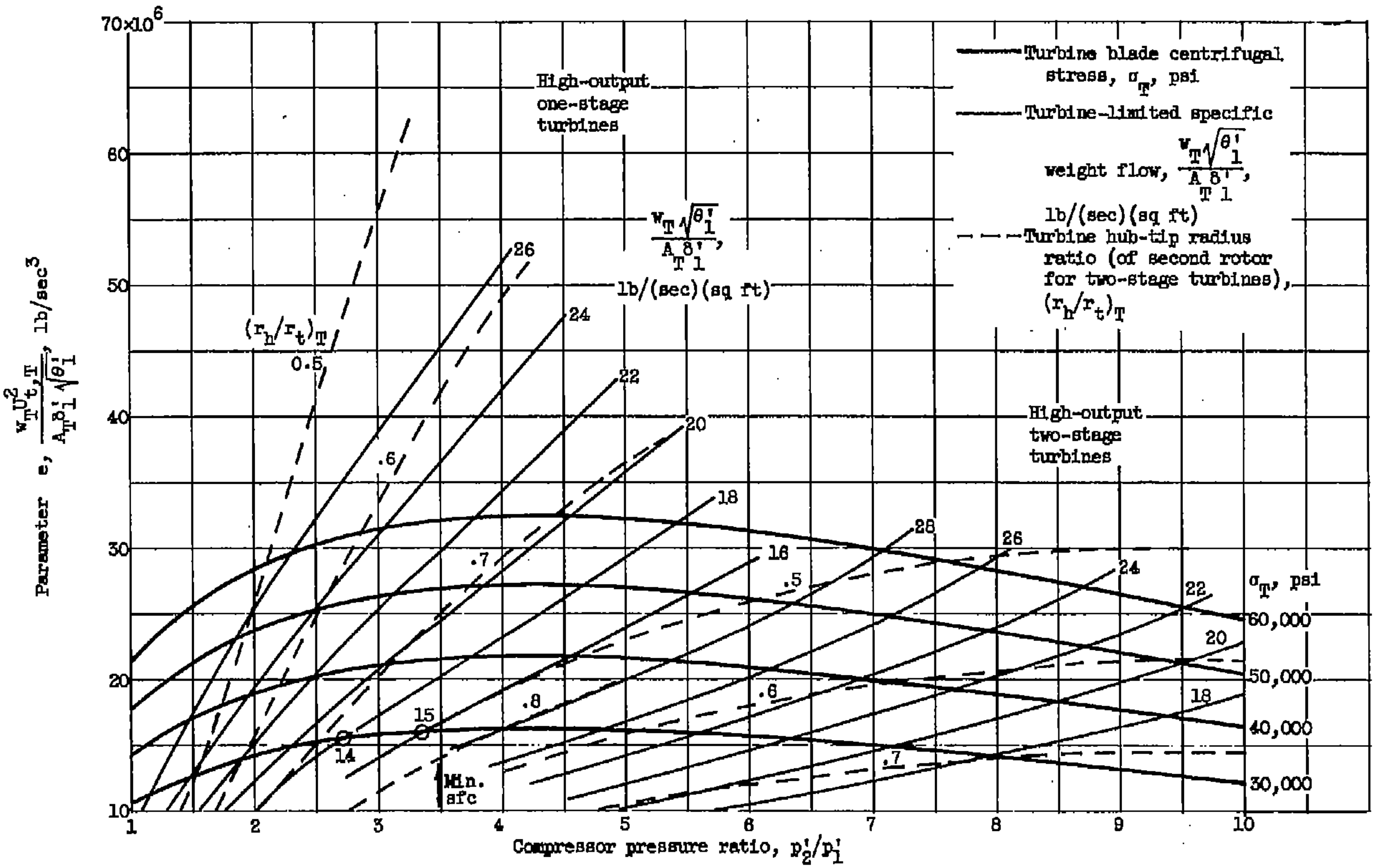
(e) Flight Mach number of 2.0 in stratosphere; engine temperature ratio, 4.0; maximum turbine relative Mach number, 0.8 for one-stage turbines, 0.6 for two-stage turbines.

Figure 4. - Continued. Turbine charts.



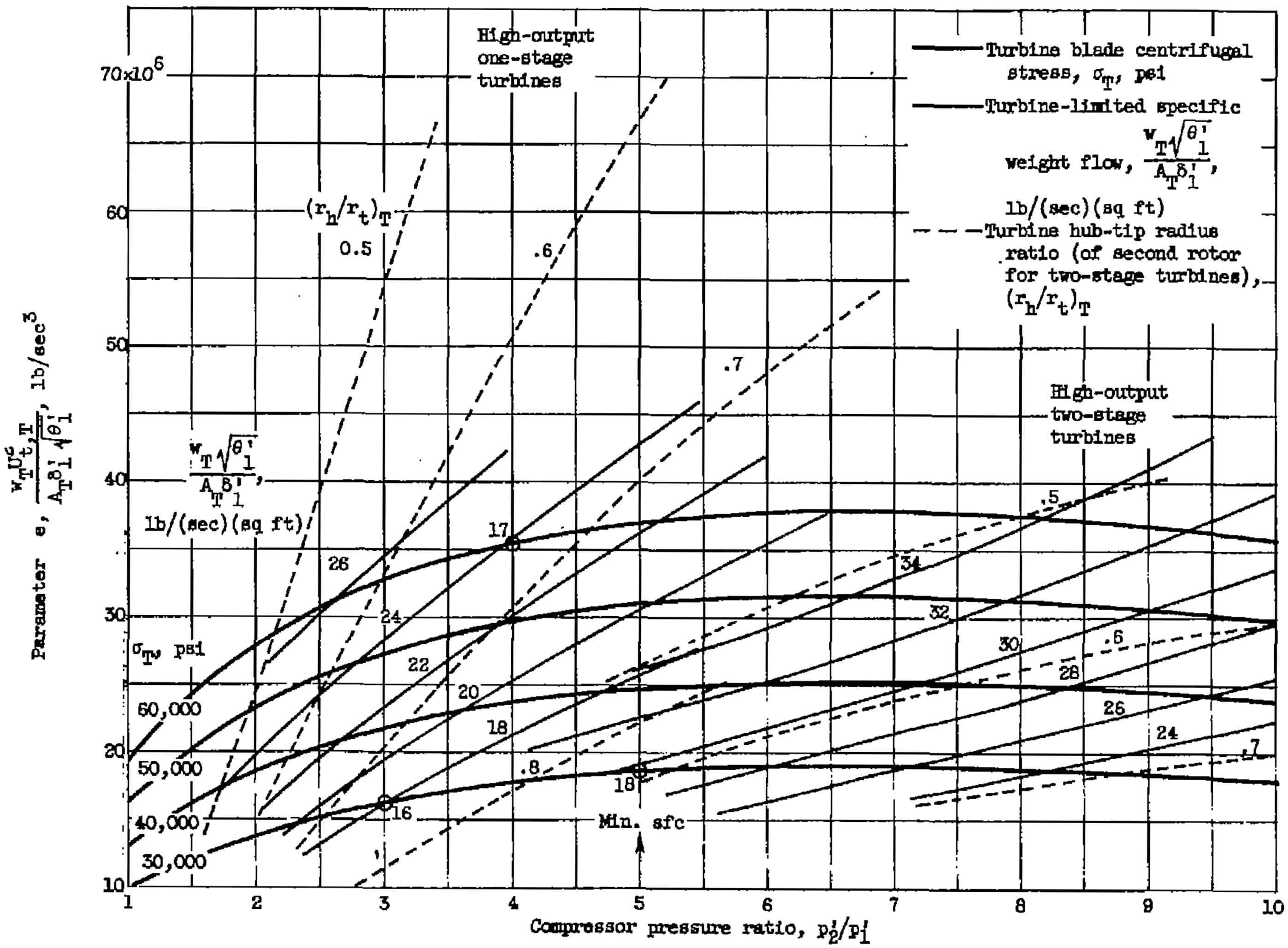
(f) Flight Mach number of 2.8 in stratosphere; engine temperature ratio, 2.0; maximum turbine relative Mach number, 0.8.

Figure 4. - Continued. Turbine charts.



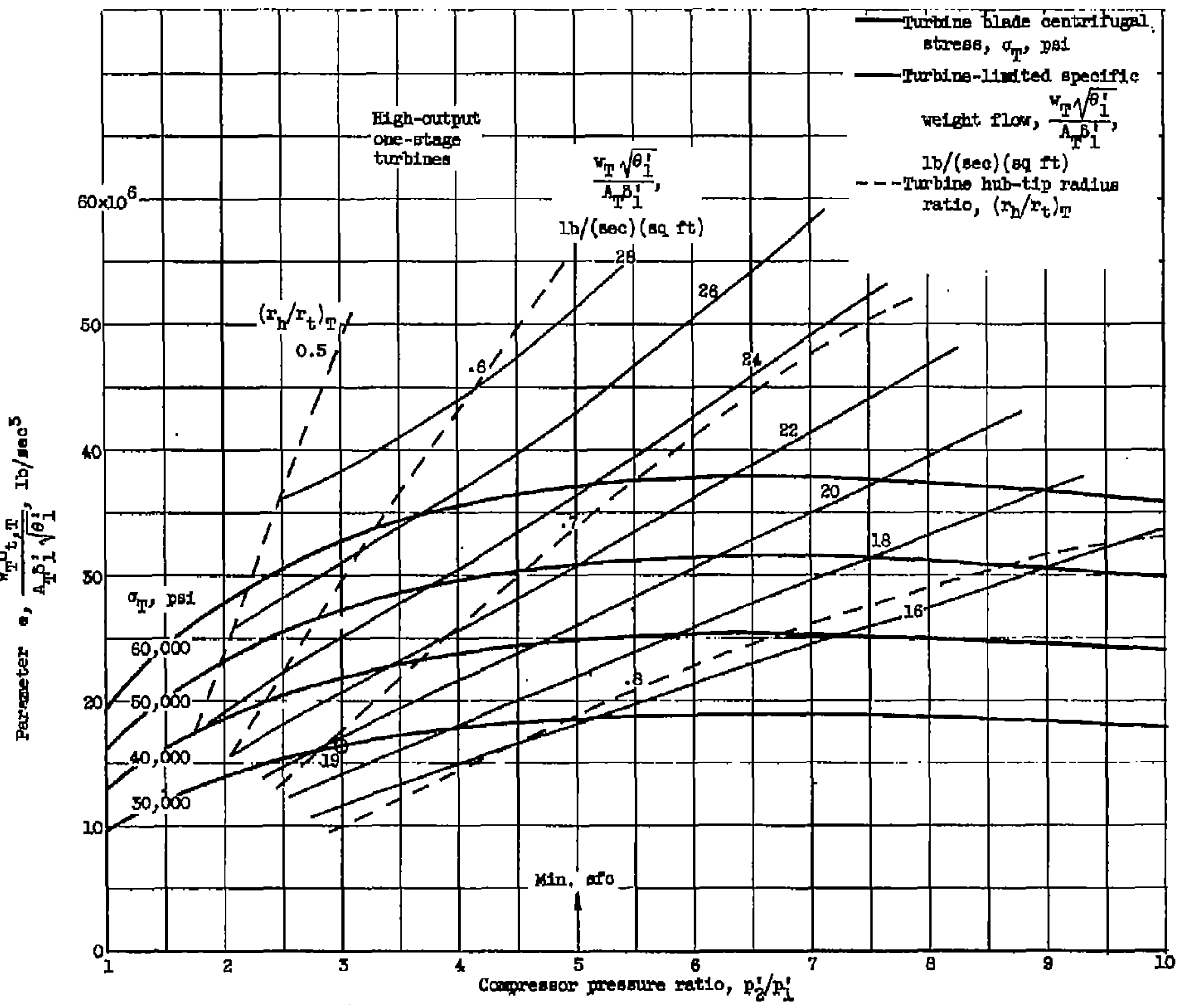
(g) Flight Mach number of 2.8 in stratosphere; engine temperature ratio, 2.5; maximum turbine relative Mach number, 0.8.

Figure 4. - Continued. Turbine charts.



(h) Flight Mach number of 2.8 in stratosphere; engine temperature ratio, 3.0; maximum turbine relative Mach number, 0.8.

Figure 4. - Continued. Turbine charts.



(i) Flight Mach number of 2.8 in stratosphere; engine temperature ratio, 3.0; maximum turbine relative Mach number, 1.0.

Figure 4. - Concluded. Turbine charts.

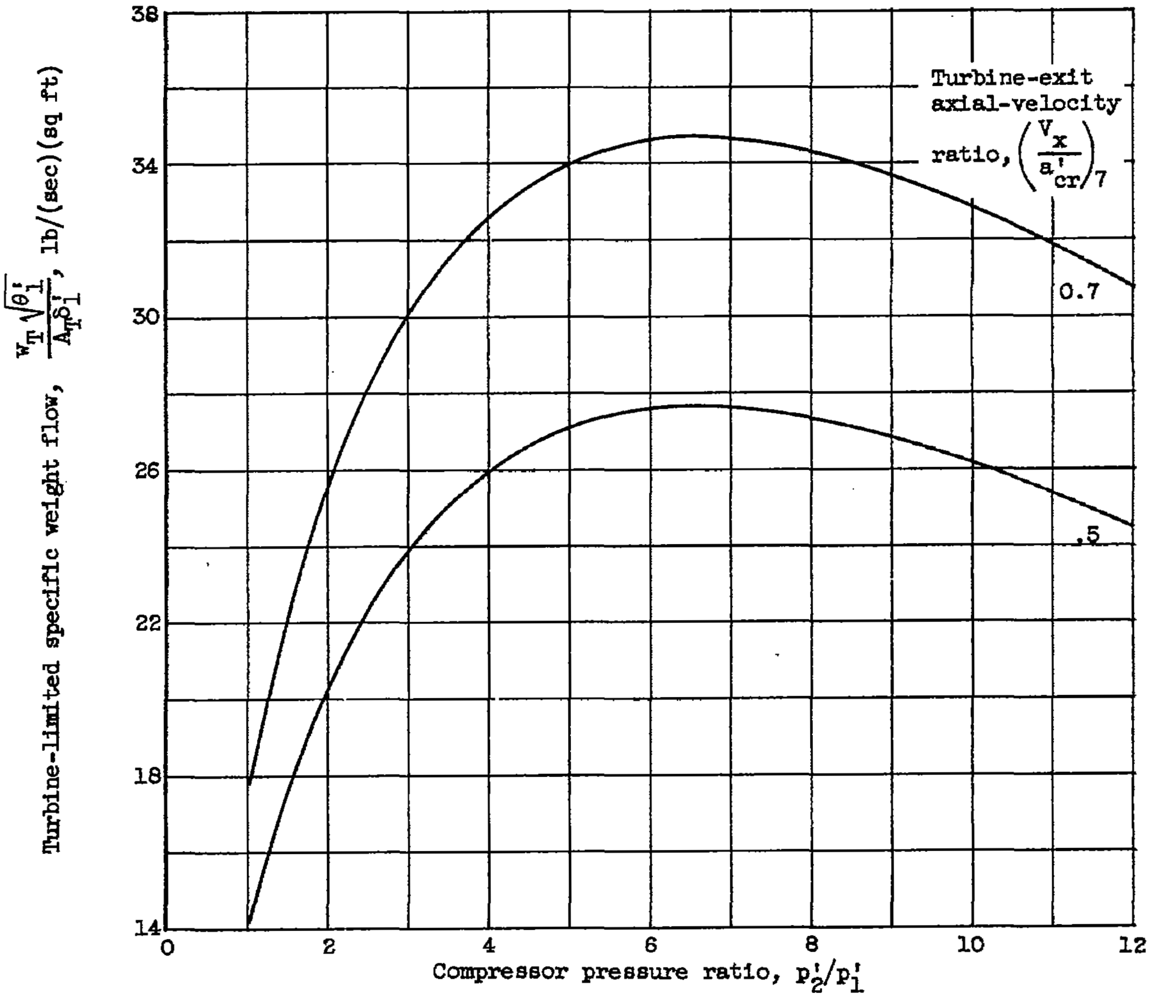


Figure 5. - Effect of turbine-exit axial-velocity ratio on turbine-limited specific weight flow. Turbine hub-tip radius ratio, 0.5; engine temperature ratio, 3.0.

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